Representing the Solution Process by Graphing
By – Lora Smith and Lacey Beshearse

Activity Overview

1. In this activity, students will explore the relationships in equations. Students will validate inquiries by graphing expressions from both sides of an equation. Students will rationalize the characteristics of graphing equations.
2. At the Pre-Algebra level, this activity can be used to compare equations and describe characteristics of the graphs.
3. At the Algebra 1 level, this activity can be used to visually compare and contrast both sides of an equation to show equality. This activity can also be used to foster critical thinking skills in secondary students.

Concepts

NCTM Standards:

In grades 9-12 all students should-

- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency-mentally or with paper and pencil in simple cases and using technology in all cases
- Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations

TN Algebra 1 Standards:

Course Level Expectation:

CLE 3102.1.4 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and to communicate solution strategies.

CLE 3102.1.7 Use technologies appropriately to develop understanding of abstract mathematical ideas, to facilitate problem solving, and to produce accurate and reliable models.

CLE 3102.3.4 Solve problems involving linear equations and linear inequalities.

Checks for Understanding:

3102.1.8 Recognize and perform multiple steps in problem solving when necessary.

3102.1.11 Use manipulatives to model algebraic concepts.

3102.1.12 Create and work flexibly among representations of relations (including verbal, equations, tables, mappings, graphs).

3102.1.13 Change from one representation of a relation to another representation, for example,

change from a verbal description to a graph.
3102.1.15 Apply arithmetic concepts in algebraic contexts.
3102.1.19 Recognize and practice appropriate use of technology in representations and in problem solving.
3102.2.1 Recognize and use like terms to simplify expressions.
3102.2.2 Apply the order of operations to simplify and evaluate algebraic expressions.
3102.3.11 Solve multi-step linear equations with one variable.
3102.3.13 Solve multi-step linear inequalities with one variable and graph the solution on a number line.
3102.3.23 Determine the graph of a linear equation including those that depict contextual situations.
3102.3.24 Interpret the changes in the slope-intercept form and graph of a linear equation by looking at different parameters, m and b in the slope-intercept form.
3102.3.25 Find function values using f(x) notation or graphs
3102.3.26 Graph linear inequalities on the coordinate plane and identify regions of the graph containing ordered pairs in the solution.
3102.3.28 Solve systems of linear equations graphically, algebraically, and with technology.

State Performance Indicators:

SPI 3102.3.5 Write and/or solve linear equations, inequalities, and compound inequalities including those containing absolute value.
SPI 3102.3.6 Interpret various relations in multiple representations.
SPI 3102.3.8 Determine the equation of a line and/or graph a linear equation.

Teacher Preparation
Before the activity, the teacher should pre-load the activity file *solution process by graphing.tns* to the student handheld devices. This can be done via Connect-to-Class software, TI-Nspire computer link software, or by using link cables with the handhelds.

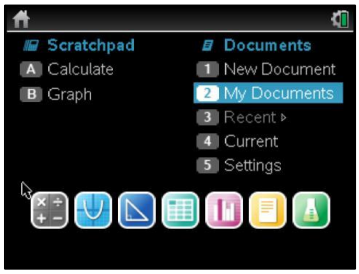
Classroom Management Tips
1. This activity can be done by the students using the pre-made document file or by them creating various parts of the file as chosen by the teacher.
2. This activity is intended to the student-centered with the students working in small cooperative groups.

TI-Nspire Applications
- ✓ Notes
- ✓ Lists & Spreadsheets
- ✓ Graphs

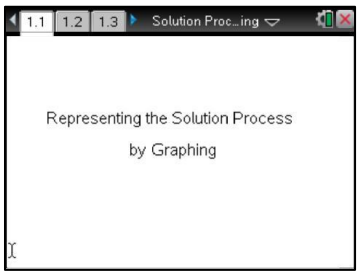
Step-by-Step Directions

Steps:

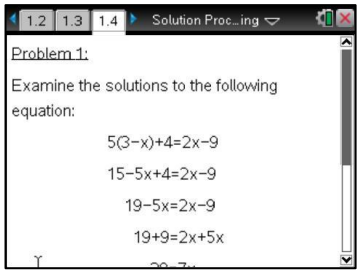
1. From the home screen, choose My Documents and navigate to the appropriate folder containing the .tns file *solution process by graphing.* Highlight the file and press ___. Choose whether or not to save changes to any previous document.



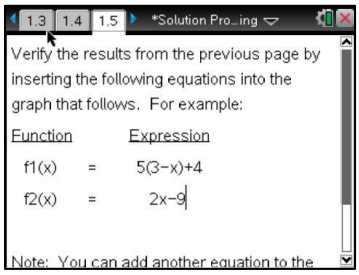
2. Page 1.1 is the title page for the activity. Press ctrl and arrow to navigate through the document. Pressing ___will give you a thumbnail view of all pages in the document. Pages 1.2 and 1.3 list standards addressed in this activity.



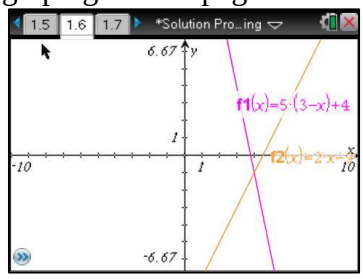
3. 1.4 introduces the first of three problems in this activity. Students are prompted to verify the results on the next page.



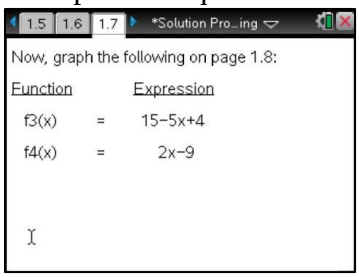
4. Students may notice that the first two expressions are from the initial equation.



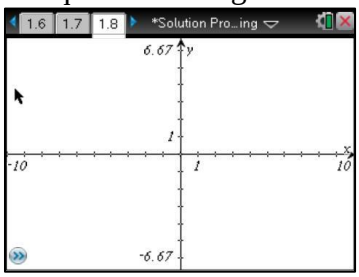
5. Students will then graph the expressions from Page 1.5 on the graph given on page 1.6.



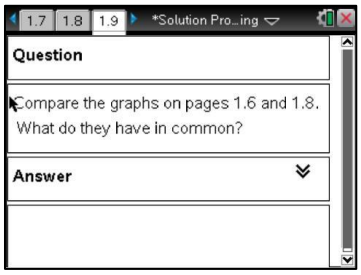
6. Students will be prompted to graph the expressions from the first step in the equation.



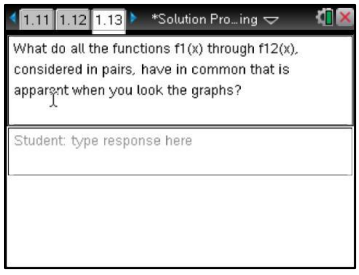
7. Students will then graph the expressions from the first step of the equation on Page 1.8.



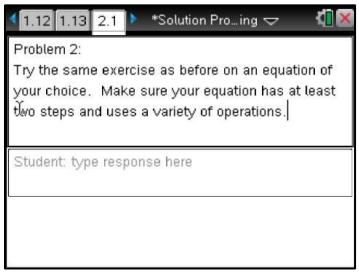
8. Students can answer the various questions within the document by pressing the down arrow until they get to the answer section of the page. Page 1.11 gives instructions to graph the remaining expressions from the equation. Students will be able to move a line's label so that all labels can be seen.



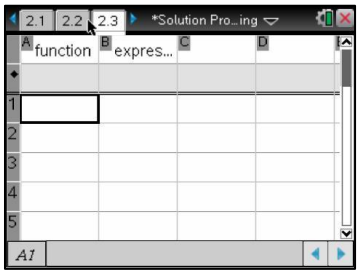
9. Following the graph, students are asked to address the relationship between all sets of expressions. Students will be able to refer all three graphs previously made to answer a follow-up question. Hopefully, students will make the statement that all the pairs of functions have the same *x*-coordinate of their points of intersection.



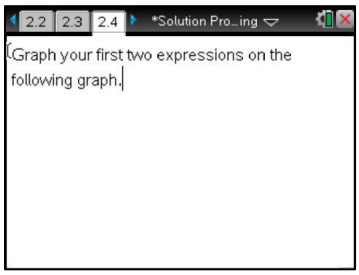
10. Problem 2 is presented on 2.1. Students are asked to create their own equation with multiple steps and various operations.



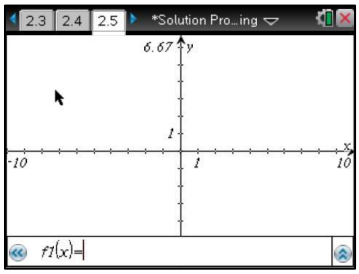
11. Students are then asked to solve each step in the equation using the spreadsheet on Page 2.3. Each part of the solution process should be completed in a separate cell.



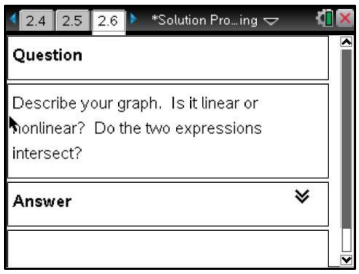
12. Students are then prompted to graph the first two expressions listed in their spreadsheet.



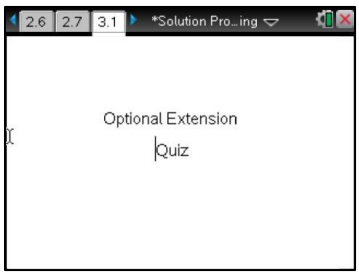
13. Students should graph the expressions on Page 2.5.



14. Following the graph, students are prompted to answer a question concerning the characteristics of the graph itself. Students are then asked to graph their remaining expressions on the same graph on page 2.5.



15. If time allows, Pages 3.1 to 3.8 can be used as an extension quiz. Students will use the previous problems as a guide for the quiz. Similar strategies will be used.



Assessment and Evaluation

- ✓ The teacher can collect student files using Connect-to-Class software or TI-Nspire Computer Link software.
- ✓ Sample Answers to questions in .tns file:

  Q: Compare the graphs on pages 1.6 and 1.8. What do they have in common?

  A: Students might say that the graphs have the same intersection. Students also may say that the lines have the same slope in both graphs.

  Q: Based on your comparison, what do you expect the remaining expressions to have in common?

  A: Students may say that they expect all graphs to have the same intersection and all lines from the same side of the equation will look the same.

  Q: What do all the functions f1(x) through f12(x), considered in pairs, have in common that is apparent when you look at the graphs?

  A: Students might conjecture that all the pairs of functions have the same x-coordinate of their points of intersection.

  Q: What do all the pairs of lines have in common?

  A: All the pairs of functions have the same x-coordinate of their points of intersection.

  Q: Describe your graph. Is it linear or nonlinear? Do the two expressions intersect?

  A: The graph is linear. The two expressions do intersect.

  Q: Explain whether you think the commonality in the pairs of functions will always occur.

  A: The pairs of equations will always have a common point of intersection when the solutions involve linear expressions.

  Q: Each pair of functions used in the exercise is a system of linear equations. What do you think is necessary in order for two systems of equations to be equivalent?

  A: (a.) The graphs appear to intersect at the point with coordinates (3, 6).
  (b.) The solution is not really a solution because x cannot be equal to 3. If $x = 3$, then the left side of the equation contains a division by 0.

  Q: If you graphed the functions f1(x) and f2(x), where do you think the lines obtained intersect?

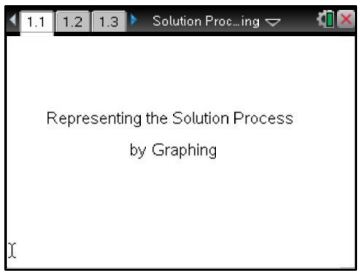
  A: Students may conjecture that the graphs will intersect at a given point. Students also may conjecture that the graphs will intersect at the solution of the system of equations.

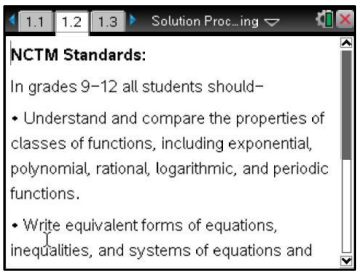
  Q: What is the meaning of this point of intersection?

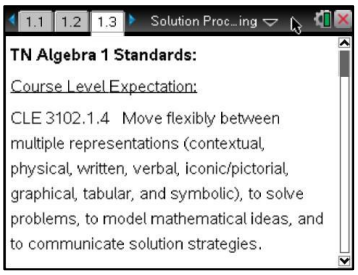
  A: Students should convey that the point of intersection is the solution of the system of equations. The two expressions of each step in solving the equation make a system of equations.

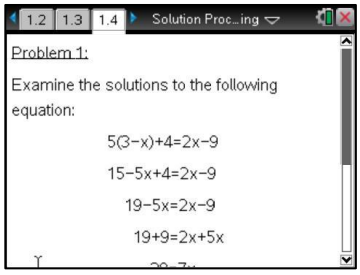
Student TI-Nspire Document

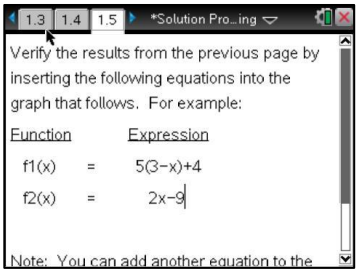
Solution process by graphing.tns

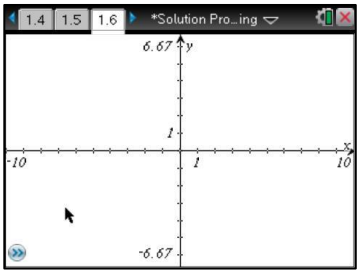


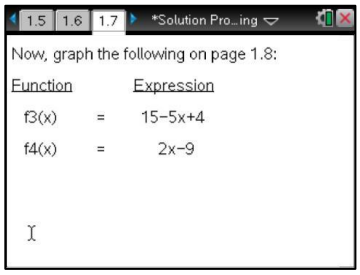


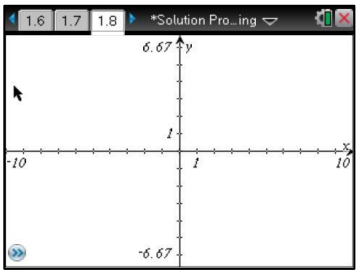


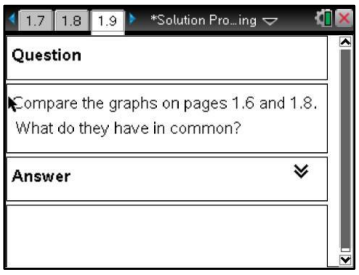


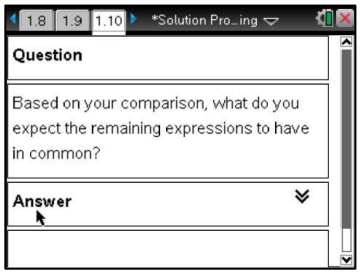


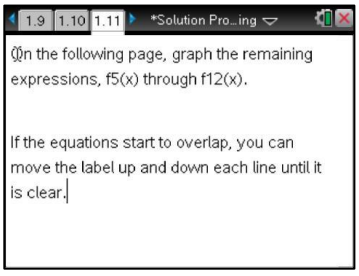


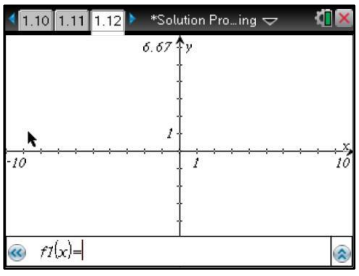


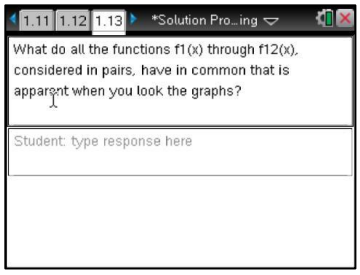


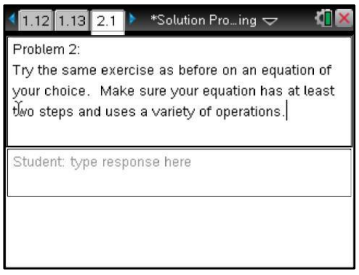


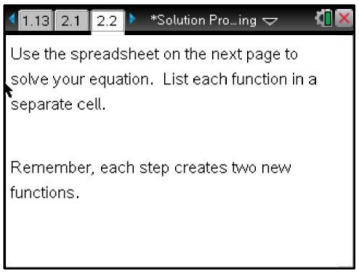


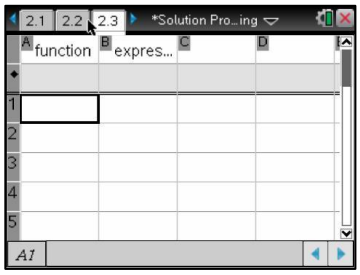


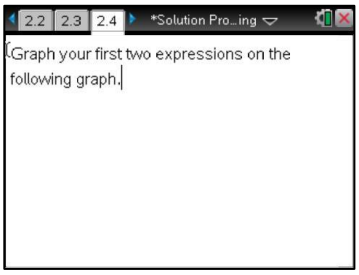


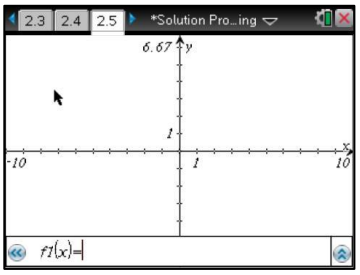


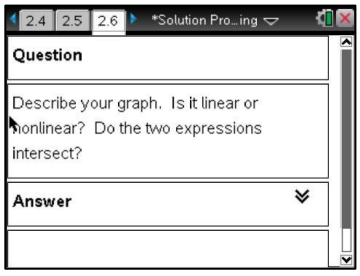


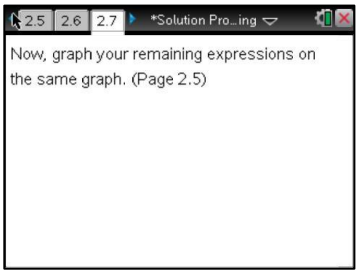


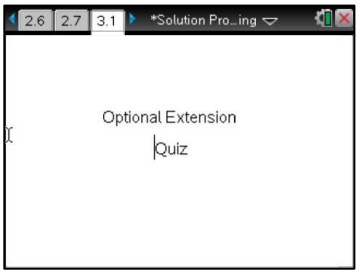

2.1 2.2 2.3 *Solution Pro...ing
A function B expres... C D
1
2
3
4
5
A1

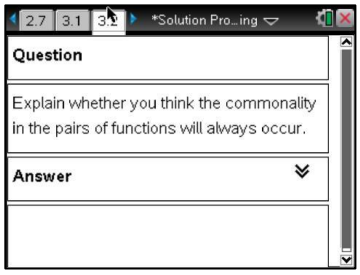

2.2 2.3 2.4 *Solution Pro...ing
Graph your first two expressions on the following graph.

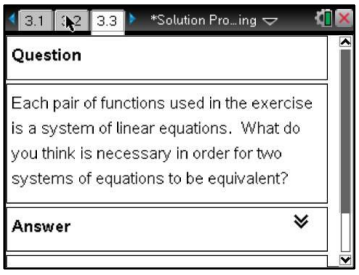

2.3 2.4 2.5 *Solution Pro...ing
6.67
y
1
x
-10
1
10
f1(x)=

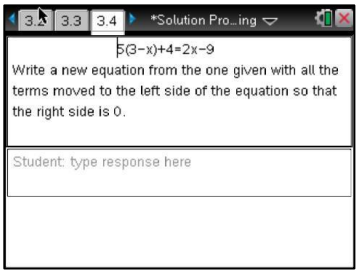

2.4 2.5 2.6 *Solution Pro...ing
Question
Describe your graph. Is it linear or nonlinear? Do the two expressions intersect?
Answer

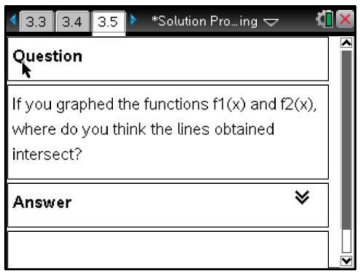

2.5 2.6 2.7 *Solution Pro...ing
Now, graph your remaining expressions on the same graph. (Page 2.5)

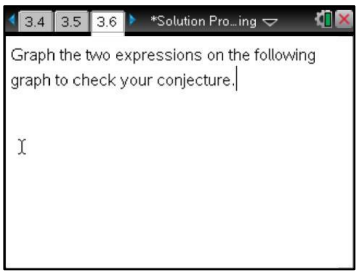

2.6 2.7 3.1 *Solution Pro...ing
Optional Extension
Quiz

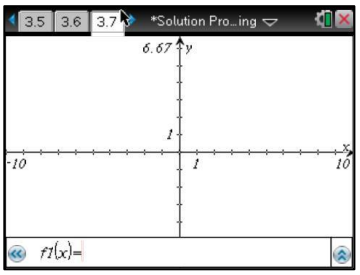

2.7 3.1 3.2 *Solution Pro...ing
Question
Explain whether you think the commonality in the pairs of functions will always occur.
Answer

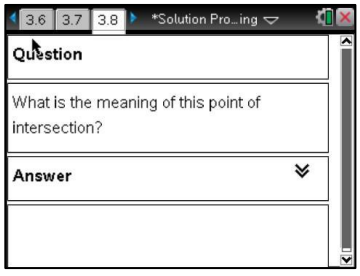

3.1 3.2 3.3 *Solution Pro...ing
Question
Each pair of functions used in the exercise is a system of linear equations. What do you think is necessary in order for two systems of equations to be equivalent?
Answer

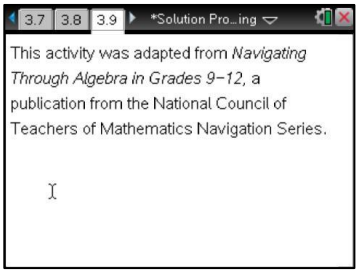

3.3 3.4 *Solution Pro...ing
5(3−x)+4=2x−9
Write a new equation from the one given with all the terms moved to the left side of the equation so that the right side is 0.
Student: type response here


3.3 3.4 3.5 *Solution Pro...ing
Question
If you graphed the functions f1(x) and f2(x), where do you think the lines obtained intersect?
Answer


3.4 3.5 3.6 *Solution Pro...ing
Graph the two expressions on the following graph to check your conjecture.


3.5 3.6 3.7 *Solution Pro...ing
6.67
y
1
x
-10
1
10
f1(x)=


3.6 3.7 3.8 *Solution Pro...ing
Question
What is the meaning of this point of intersection?
Answer


3.7 3.8 3.9 *Solution Pro...ing
This activity was adapted from *Navigating Through Algebra in Grades 9–12*, a publication from the National Council of Teachers of Mathematics Navigation Series.

This activity was adapted from *Navigation Through Algebra in Grades 9-12, Chapter 4: Expanding Understanding of Algebraic Equivalence or Identity*, a publication from the National Council of Teachers of Mathematics Navigation Series.