Moveable Lines

Teachers Notes & Answers

7 8 9 10 **11** **12**

TI-Nspire™ Investigation Student 60 min

Aim

- Determine solutions of literal equations and general solution of equations involving a single parameter
- Determine solutions of simple systems of simultaneous linear equations, including consideration of cases where no solution or infinite number of possible solutions exist (geometric interpretation only required for two equations in two variables)

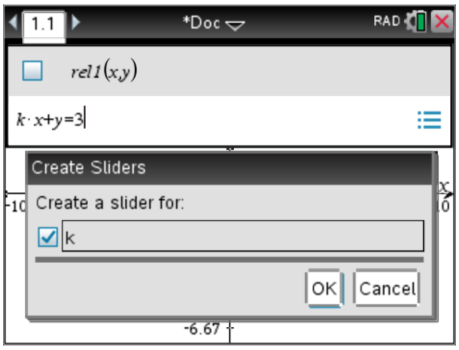
Instructions

Consider the following pair of linear simultaneous equations where k is a constant.

$$kx + y = 3$$
$$2x + (k+1)y = 6$$

Enter the equations using the relational graphing type.

Since k has not been defined anywhere a slider can automatically be created, select OK to create the slider.



A multiplication sign must be entered between two alpha characters such as the k and x in the first equation.

With the cursor over the slider, press **Ctrl** + **Menu**, this activates the contextual menu [right mouse click] for the slider; select settings and set the slider as shown opposite.

Enter the second equation in relation 2; notice that since a slider has already been established for k you are not prompted for another. The one slider will control the value of k in both equations.

Question: 1

Use the slider to vary the value of k and observe what happens on the screen, record your observations. For $-5 \le k \le 5$ students will see a range of values that provide 1 solution, when $k = 1$ they will see 'infinitely many solutions and for $k = -2$, no solutions. It should however be noted that these are approximate solutions that need to be verified algebraically. Students may also not that the gradient of both lines is affected by k.

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Question: 2

Use algebra to find the solution to the set of simultaneous equations for each of the following cases:

a) $k = 0$

Equations:

$$2x + y = 6$$

$$y = 3$$

Students may also acquire these equations on a calculator application:

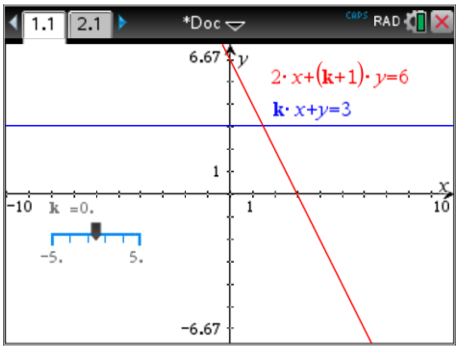
rel1(x,y)

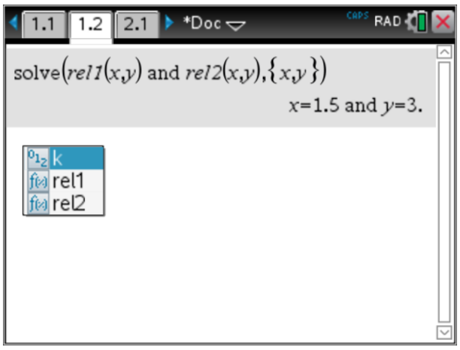
While students can use the solve command, this problem is a nice illustration that by-hand calculations can save time:

$$2x + (3) = 6 \qquad (1.5, 3)$$

$$x = 1.5$$

Gradients: $m_1 = -2$ and $m_2 = 0$





b) $k = -1$

Equations:

$$2x = 6$$

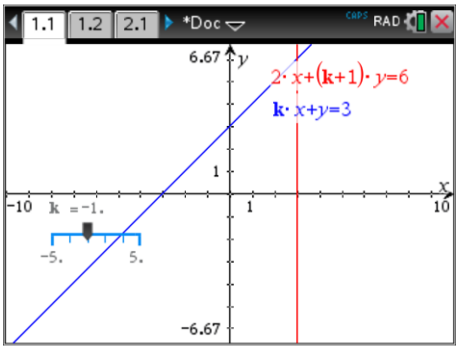
$$-x + y = 3$$

Once again, students can use the solve command, graphical feedback should encourage students to look carefully at the equation … and then consider solutions by-hand:

$$-(3) + y = 3 \qquad (3, 6)$$

$$y = 3$$

Gradients: $m_1 = \infty$ and $m_2 = 1$



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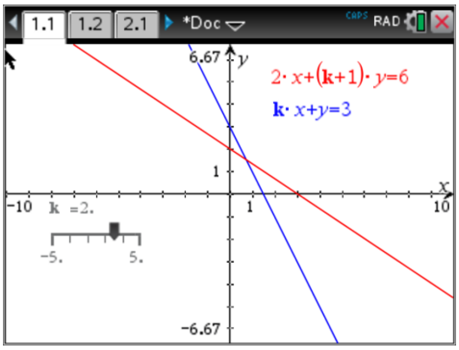
c) $k = 2$

Equations:

$$2x + 3y = 6$$

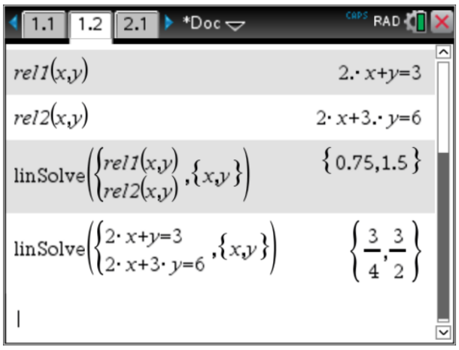
$$2x + y = 3$$

By hand elimination would provide a fast solution for able students. In addition to 'solve' students may use **solve system of equations** and use either rel1(x,y) or enter the equation directly. Notice the difference in the answers produced by the CAS.



Point of intersection: $\left(\frac{3}{4}, \frac{3}{2}\right)$

Gradients: $m_1 = -2$ and $m_2 = -\frac{2}{3}$



Sketch the corresponding straight lines; indicate the point of intersection and gradient for line for each value of k above. Check your CAS calculator.

Question: 3

By consideration of the gradients in the previous question, determine all the values of k for which:

a) There is more than one solution

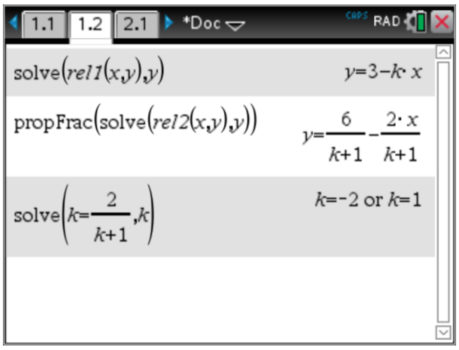
For a pair of linear equations to have: "more than one solution" can only mean one thing: 'infinite' solutions, the lines must be exactly the same.

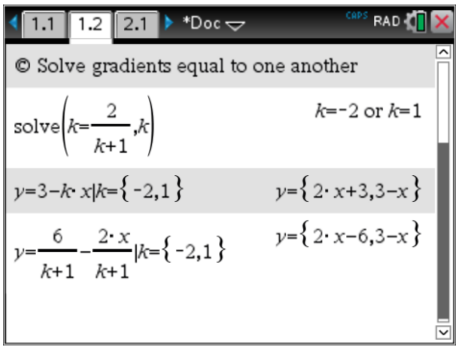
The gradient and y-intercept will be the same for both equations.

If solve command is used in Problem 1 the variable k will take on the slider value. The Delete Variable command (DelVar) can be used to remove the variable; alternatively students may insert a New Problem.

There are two values of k that produce the same gradient: k = -2 or k = 1.

Substitution of k into the two linear equations shows that when k = 1 the two equations are the same.





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b) There are no solutions

Depending on the approach to Part A, students may deduce the answer to part B using their results. (Refer results for Part A)

Graphically / Geometrically: k effects the gradient of both graphs, there are two instances where the gradients are the same meaning the lines would be parallel. In the case however when k = 1, the lines are actually the same resulting in all points being the same, infinitely many solutions.

Teacher Note: "When k = -2 the two lines have so much in common … it's a shame they'll never meet."

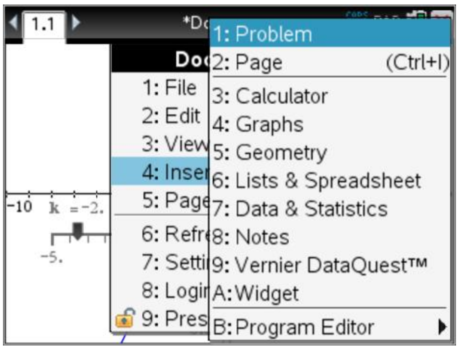
Explain graphically (or geometrically) what this means in each case.

Instructions

Insert a Graph Application in a New Problem in this document.

Doc > Insert > Problem

Enter the following equations using the relation graphing tool and add a slider as before.



$$x - y = k$$
$$x + ky = 6$$

Question: 4

Use the slider to vary the value of k and observe what happens on the screen, record your observations.

Students may write down similar comments to Question 1, however for $-5 \le k \le 5$ they will see the situation where the lines are parallel and also where they have one point of intersection. They will not see the situation where the lines are the same. Students may explore changing the slider settings … this is an illustration of the power of algebra. The slider provides a visual, but it is not a 'proof' that the lines cannot be the same.

Question: 5

Use algebra to find the solution to the set of simultaneous equations for each of the following cases:

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a) $k = 0$

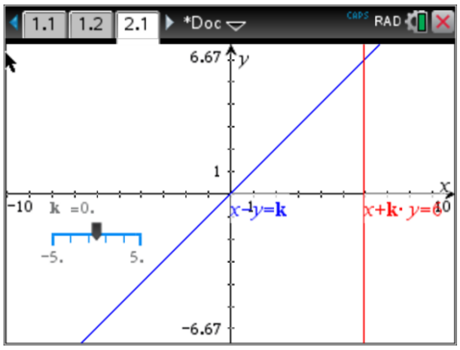
Equations:

$y = x$

$x = 6$

Point of intersection: $(6,6)$

Gradients: $m_1 = 1$ and $m_2 = \infty$



b) $k = -1$

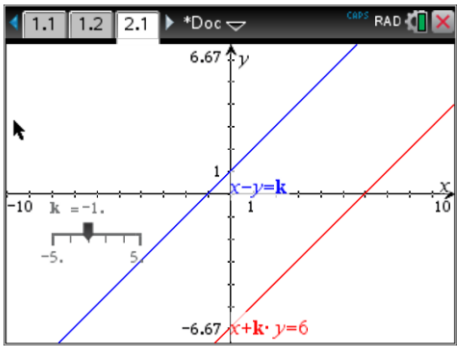
Equations:

$x - y = -1$

$x - y = 6$

Point of intersection: Parallel Lines

Gradients: $m_1 = 1$ and $m_2 = -\dfrac{1}{2}$



c) $k = 2$

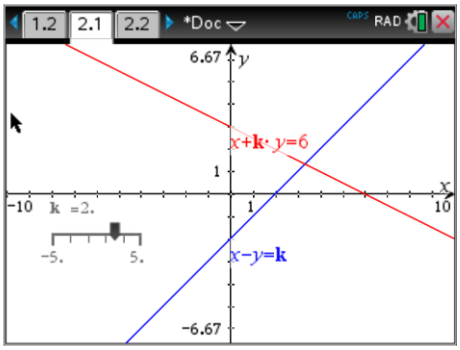
Equations:

$x - y = 2$

$x + 2y = 6$

Point of intersection: $\left(\dfrac{10}{3}, \dfrac{4}{3}\right)$

Gradients: $m_1 = 1$ and $m_2 = 1$



Sketch the corresponding straight lines; indicate the point of intersection and gradient for line for each value of k above. Check your CAS calculator.

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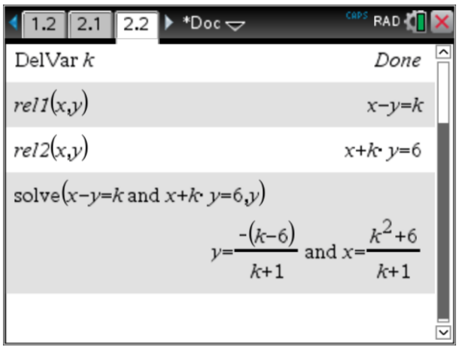
Question: 6

Explain why it is not possible to have infinitely many solutions for this set of equations.

For this pair of equations there is only one value of k for which the lines are ‘parallel’. The expression for the points of intersection shows that this occurs when k = -1. The answer to 5B shows that this is when the lines are parallel, therefore there are no values of k that will produce infinitely many solutions.



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