Jet Plane

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Abstract: This activity is an application of differentiation involving related rates. Students set up equations relating the variables and then differentiate them with respect to time to find the rate a distance is changing and an angle is changing. They use the symbolic capacity of their calculator and calculus to understand the situation better.

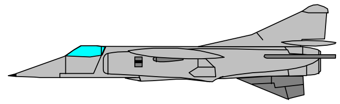
NCTM Principles and Standards:
Algebra standards
a) Understand patterns, relations, and functions
b) generalize patterns using explicitly defined and recursively defined functions;
c) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
d) use symbolic algebra to represent and explain mathematical relationships;
e) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
f) draw reasonable conclusions about a situation being modeled.

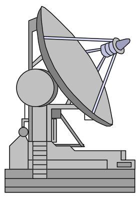
Geometry standards: Analyze characteristics and properties of two- and three-dimensional geometric shapes and mathematical about geometric relationships
Problem Solving Standard build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.
Reasoning and Proof Standard
a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;
d) select and use various types of reasoning and methods of proof.

Representation Standard : use representations to model and interpret physical, social, and phenomena.

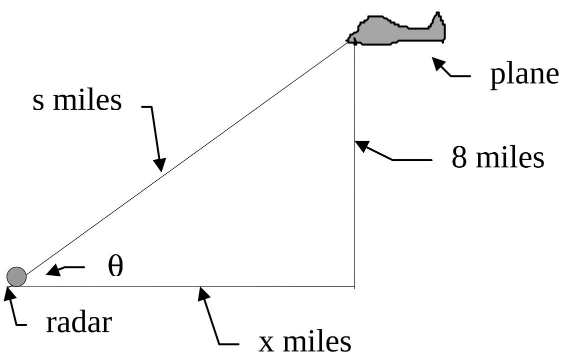
Key topic: Applications of Derivative - related rates.

Degree of Difficulty: moderate
Needed Materials: TI-89 calculator



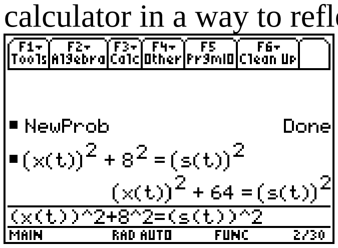


Situation: An airplane flies directly over a radar installation at 300 mph. Its altitude 8 miles and it is flying on level flight toward the station. Let's examine the rate at which the distance from the airplane to the radar station is changing and the rate at which the angle of elevation from the radar station to the plane is changing when the plane's horizontal distance to the radar station is 5 miles.

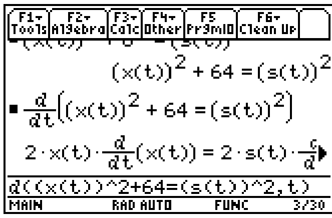


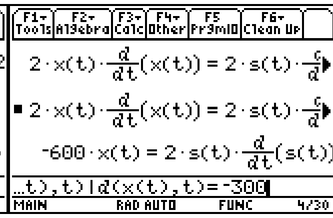
Let x represent the horizontal distance from the plane to the radar station, s represent the actual distance, and θ the angle of elevation of the plane from the radar station as shown in the above diagram.

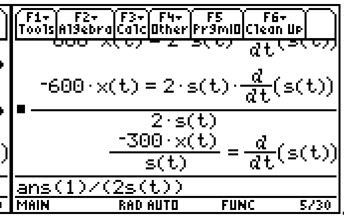
We then have $x^2 + 8^2 = s^2$ and $\tan\theta = 8/x$. We know that $\frac{dx}{dt}$ = -300 mph (as the plane is flying toward the station). Both x and s are functions of t, so we must enter them in the calculator in a way to reflect that fact:



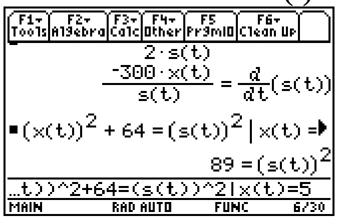
We can use this information to compute $\frac{ds}{dt}$. We first take the derivative of $x(t)^2 + 8^2 = s(t)^2$ with respect to t and then solve that equation for $\frac{ds}{dt}$ and use the fact that $\frac{dx}{dt}$ = -300

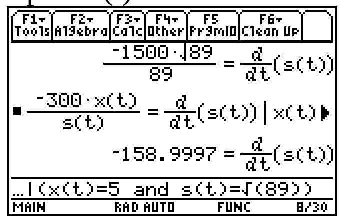




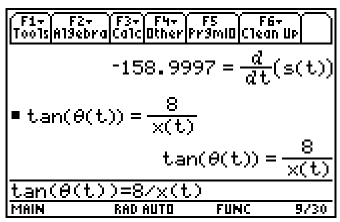


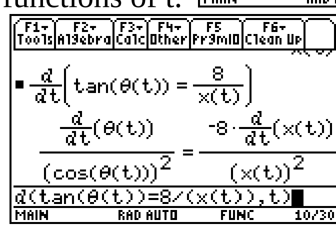
. We know that we wish to examine the situation when the plane's horizontal distance to the radar station is 5 miles so x(t) = 5. We can compute s(t) at that moment:

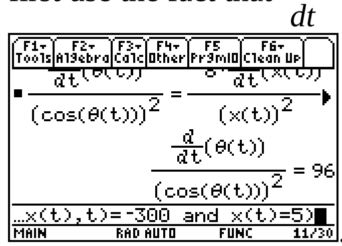


so s(t) = $\sqrt{89}$


and therefore $\frac{ds}{dt}$ ~ -159 mph.

To find $\frac{d\theta}{dt}$ we must enter the relationship between the variables to express the fact that both θ and x are functions of t.


As before we can differentiate with respect to t:


. We can first use the fact that $\frac{dx}{dt}$ = -300 and that we are interested in the situation when x(t) = 5:


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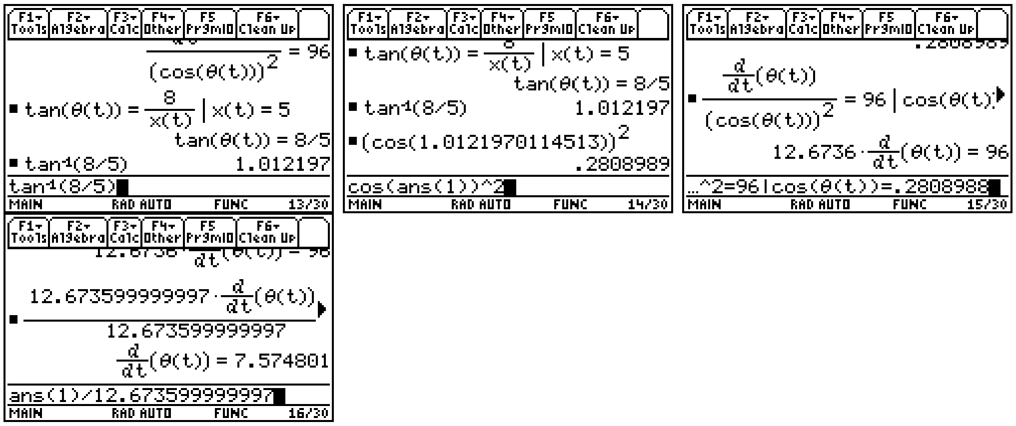
To find cos(θ(t)), we go back to the equation relating θ and x: tan θ = 8/x. We first calculate the value of tan(θ(t)), then θ(t), and then (cos (θ(t))^2 and use those to find $\frac{d\theta}{dt}$:



So our conclusion is that the plane is getting closer to the radar station at the rate of 159 mph and that the angle between the plane and the horizon is changing at the rate of 7.5748 radians/hour. Let's use the unit menu of the calculator to change that last measurement into degrees/minute.