NUMB3RS Activity: Getting in Focus
Episode: "Harvest"

Topic: Ellipses   **Grade Level:** 10 - 12
Objective: Students will use the ellipse equation to determine the location of the foci.
Time: about 20 minutes
Materials: ruler, calculator

Introduction

In "Harvest," Charlie is helping Don investigate illegal kidney transplants in Los Angeles. After analyzing the trip log of the ambulance suspected of transporting the kidneys, Charlie is able to determine that the ambulance went out of its way to go to an unknown location, traveling 26 miles when the straight distance was 20 miles. Assuming the ambulance traveled in straight lines, the unknown location would lie on an ellipse with the start and end points as the foci. In this activity, students will learn how to relate the equation of an ellipse to the coordinates of its foci.

Discuss with Students

An ellipse is the set of all points in a plane where the sum of the distances from two fixed points (foci) to the edge of the ellipse is constant.

The major axis crosses through the foci of the ellipse. The minor axis is perpendicular to the major axis, and the two axes intersect at the center of the ellipse.

The equation for an ellipse centered at the origin with a horizontal major axis is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where *a* is half the length of the major axis and *b* is half the length of the minor axis.


The foci are located on the major axis, $\sqrt{a^2 - b^2}$ units on either side of the center of the ellipse.

You will want to remind students that the major axis of an ellipse can also be vertical. In this case, the equation of the ellipse (centered at the origin) is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

Student Page Answers:
1a. *Students' graphs will show an ellipse through the given points.* **1b.** *a = 11, b = 5*

1c. *Approximately (9.798, 0) and (–9.798, 0).* **2.** *Possible answers:* $\frac{x^2}{13^2} + \frac{y^2}{12^2} = 1$; *Other points*

on the ellipse could include (–13, 0), (0, –12), and (0, 12).
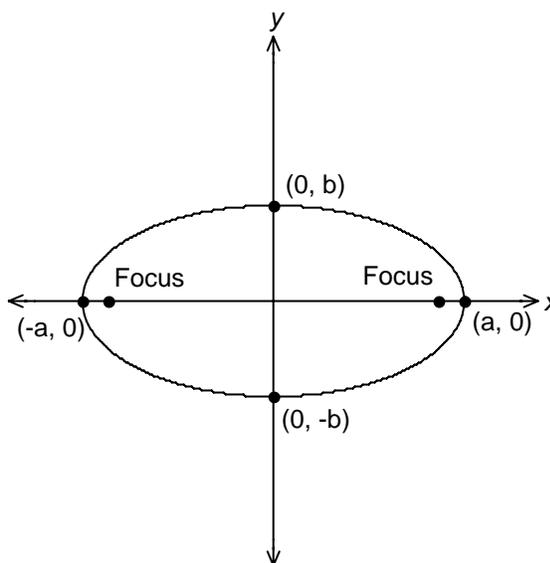
Name: _____ Date: _____

NUMB3RS Activity: Getting in Focus

In "Harvest," Charlie is helping Don investigate illegal kidney transplants in Los Angeles. After analyzing the trip log of the ambulance suspected of transporting the kidneys, Charlie is able to determine that the ambulance went out of its way to go to an unknown location, traveling 26 miles when the straight distance was 20 miles. Assuming the ambulance traveled in straight lines, the unknown location would lie on an ellipse with the start and end points as the foci.

In an ellipse, the major axis is always the longer axis, while the minor axis is the shorter axis. The major and minor axes cross at the center of the ellipse. When the center of the ellipse is located at (0, 0), the ellipse crosses the axes at the points shown below, and the foci are located as shown.
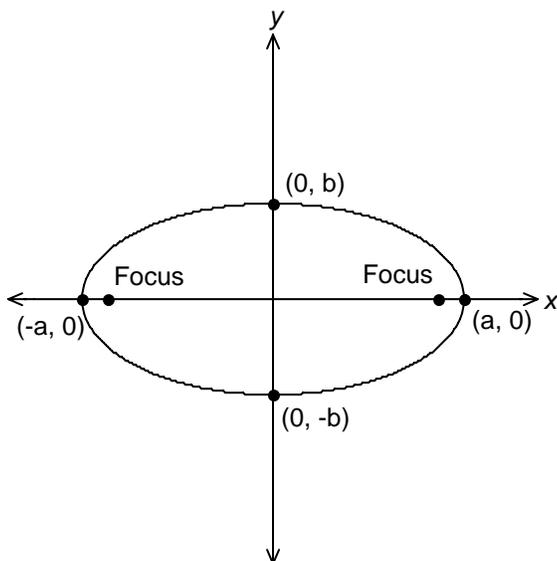

The equation for an ellipse centered at the origin with a horizontal major axis is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where *a* is half the length of the major axis and *b* is half the length of the minor axis.

1. Suppose that the following points describe some of the locations visited by the ambulance.

$$(0, -5), (1, 5), (-10, 2), (-6, 4), (-1, -5), (10, -2), (6, 4), (1, -5),$$
$$(-6, -4), (10, 2), (6, -4), (-10, -2), (11, 0), (0, 5), (-11, 0), (-1, 5)$$

a. Plot these points on the grid below. Carefully sketch the ellipse determined by the data points, and draw the major and minor axes.
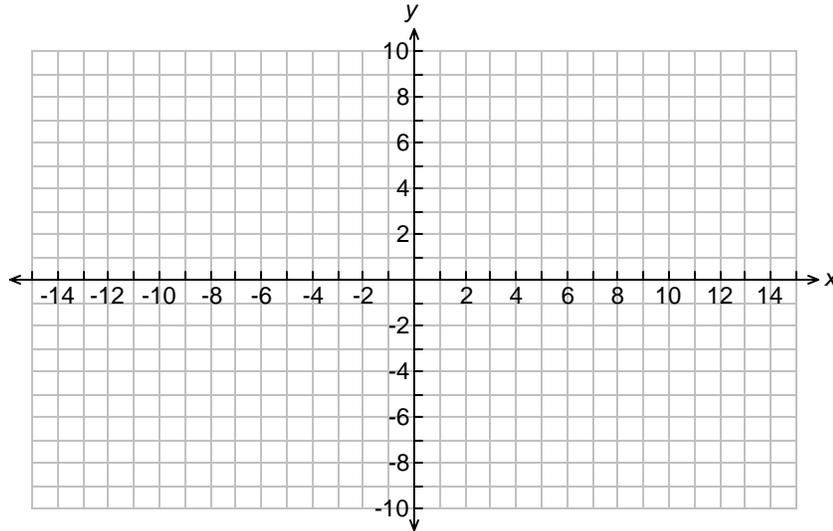

b. What are the values of *a* and *b*?_____

Charlie believes that the foci of the ellipse may give the location of the start and stop points of the ambulance. In an ellipse, the foci lie on the major axis, and are located $\sqrt{a^2 - b^2}$ units on either side of the center of the ellipse.

c. What are the coordinates of the two possible locations? That is, what are the coordinates of the foci of this ellipse? _____

2. Using his map of Los Angeles, Charlie sets up a coordinate system (in units measuring one mile) that overlays the map. Assuming the ambulance traveled in straight lines, the unknown location would lie on an ellipse with the start and end points as the foci. Charlie plots the focal points at (5, 0) and (–5, 0) and a point on the ellipse at (13, 0). Find an equation for the ellipse. Name three other points on the ellipse.
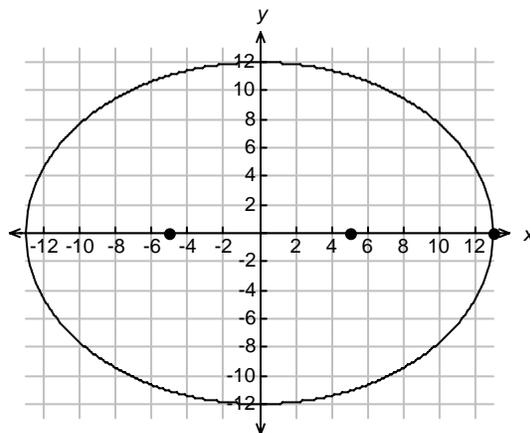
The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

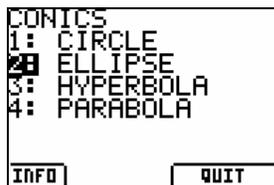# Extension: Using the Conic Graphing App

Introduction
You can also use the Conic Graphing App on the TI-84 Plus calculator to find the foci of an ellipse. This App can also draw the graph of an ellipse. This extension will give an overview of how to use this App.
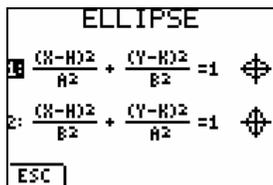
For the Student
This example will show how to use the Conic Graphing App for the ellipse in question #1 of the student page. For this ellipse, $a = 11$ and $b = 5$.
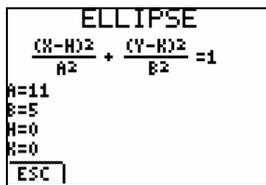
To start the App, press APPS and select **Conics**.

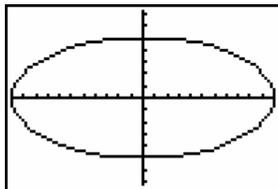|			
CONICS 1: CIRCLE 2: ELLIPSE 3: HYPERBOLA 4: PARABOLA INFO QUIT	ELLIPSE 1: $\frac{(X-H)^2}{A^2} + \frac{(Y-K)^2}{B^2} = 1$ 2: $\frac{(X-H)^2}{B^2} + \frac{(Y-K)^2}{A^2} = 1$ ESC	ELLIPSE $\frac{(X-H)^2}{A^2} + \frac{(Y-K)^2}{B^2} = 1$ A=11 B=5 H=0 K=0 ESC	ELLIPSE CENTER C=(0,0) FOCUS F1=(-9.798,0) FOCUS F2=(9.798,0) ESC
Select ELLIPSE	Select 1:	Enter 11 for A and 5 for B.	Press ALPHA ENTER

To draw the graph of the ellipse, press GRAPH.

Additional Resources
• Ellipses are a part of a family of graphs called conic sections. To learn more about conic sections, visit **http://britton.disted.camosun.bc.ca/jbconics.htm**.

• Information about how ellipses are related to planetary motion can be found at **http://csep10.phys.utk.edu/astr161/lect/history/kepler.html**.

• The Conic Graphing App can be downloaded for free from **http://education.ti.com/us/conicgraphingapp**.