APPLICATIONS OF FINITE AND INFINITE SERIES

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(Please feel free to email me questions and /or comments.)

Key Topic: Infinite Series

Abstract:

This activity is designed to show students the power of finite and infinite series. It can be used as an introduction to infinite series provided that the student is adept at using the Σ notation to denote such series. No knowledge of how to evaluate the series is necessary since the student will be shown, in great detail, how to do this using a TI-89.

Prerequisite Skills:

- Ability to represent finite and infinite series using the Σ notation.
- Ability to solve word problems

Degree of Difficulty: Easy to moderate

Needed Materials: TI-89

NCTM Principles and Standards:

- Content Standards – Algebra
  - Represent and analyze mathematical situations and structures using algebraic symbols
  - Use mathematical models to represent and understand quantitative relationships
  - Draw a reasonable conclusion about situation being modeled

- Process Standards
  - Representation
  - Connections
  - Problem Solving

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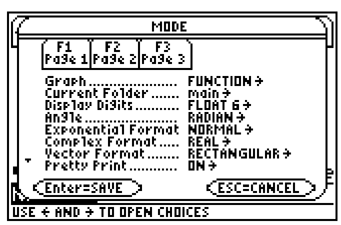
All fractions can be represented as either a finite decimal or an infinite, repeating decimal. For example, the finite decimal 0.5 is the fraction $\frac{1}{2}$, and the infinite, repeating decimal $0.\bar{3} = 0.333333\cdots$ is the fraction $\frac{1}{3}$.

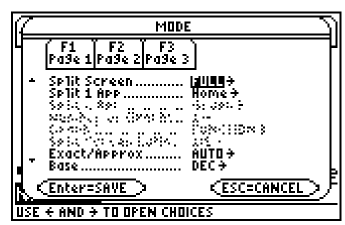
OK, so you know these values because you have come across them so many times. But do you know the fractional equivalent of $0.\bar{1}$? Those of you who remember the formula for converting infinite, repeating decimals to fractions know that the answer is $\frac{1}{9}$. But independent of whether or not you remembered this formula, let's look at how using a TI-89 to evaluating a series will help us find the answer.

Expressed as an infinite sum, $0.\bar{1} = 0.1 + 0.01 + 0.001 + 0.0001 + \cdots$. In base 10 notation this is equivalent to $0.\bar{1} = \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \frac{1}{10^4} + \cdots$. And in "summation notation" we get $0.\bar{1} = \sum_{i=1}^{\infty} \frac{1}{10^n}$. This is easy to evaluate using the TI-89. In the following we will show you how. But first we must take care of some house keeping details.

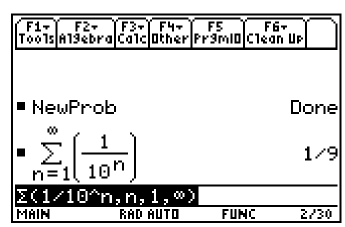
Whenever you start a new problem, clear memory by pressing [2nd][F6]**2.**

Set the first two pages of the **MODE** as shown at the right.





Enter and evaluate the infinite series by pressing [F3][4][1][÷][1][0][^][alpha] **n** [,][alpha] **n** [,][1][,] [◆][∞][)][ENTER]. **Note:** [∞] is above the [CATALOG] key.

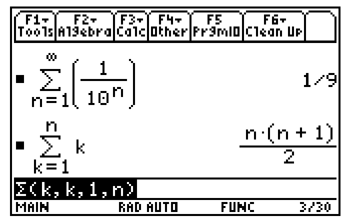


The general format for the summation notation on the TI-89 is

$$\sum(\text{expression, variable, lower limit, upper limit}).$$

In the example above, the expression was $1/10^n$, making n the variable. And the limits on the variable were from 1 to ∞.

Another fact which you may, or may not, know is that the sum of the first n counting numbers is $\frac{n(n-1)}{2}$. That is, $1+2+\cdots+n=\frac{n(n-1)}{2}$. But if you don't remember this, it's not problem. The TI-89 can find the answer for you. Here's how.



We want to find the value of $\sum_{k=1}^{n} k$. So press

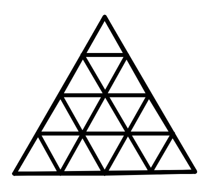
[F3][4][alpha] **k** [,][alpha] **k** [,][1][,][alpha] **n** [)][ENTER].

EXERCISES: The following problems are taken from the Calendar Section of various issues of the **Mathematics Teacher**. The first two can be found in the April, 1992 issue, and the third appeared in the December, 1991 issue.

1. Find $\frac{1}{7}+\frac{2}{7^2}+\frac{1}{7^3}+\frac{2}{7^4}+\cdots$

2. A postal employee delivered mail daily for 42 days, each day delivering 4 more letters than the day before. The total delivery for the first 24 days of the period was the same as that for the last 18 days. How many letters did the employee deliver during the whole 42 day period?

3. If you added twenty rows to the bottom of this picture, how many small triangles would you have altogether?



ANSWERS:

1. $\frac{9}{48}$

2. 12,096

3. 576