Inscribed Angles – ID: 9687

By Michele Patrick

Time required
25 minutes

Activity Overview

In this activity, students use animation to discover that the measure of an inscribed angle is half the measure of its intercepted arc, that two angles that intercept the same, or congruent, arcs are congruent, and that an angle inscribed in a semi-circle is a right angle. They then discover that the opposite angles of a quadrilateral inscribed in a circle are supplementary.

Concepts

- *Inscribed angles in circles*
- *Diameters and semi-circles*
- *Inscribed quadrilaterals*

Teacher Preparation

This activity is designed to be used in a high school geometry classroom.

- *This activity assumes that students are familiar with the terms **inscribed angle** and **intercepted arc**. If they do not know these terms, you can introduce them prior to beginning the activity. This activity also assumes knowledge of diameters, semi-circles, arcs, and quadrilaterals. Students should know that a complete circle measures 360°.*
- *The screenshots on pages 2–4 demonstrate expected student results. Refer to the screenshots on page 5 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student and solution .tns files and student worksheet, go to education.ti.com/exchange and enter "9687" in the quick search box.***

Classroom Management

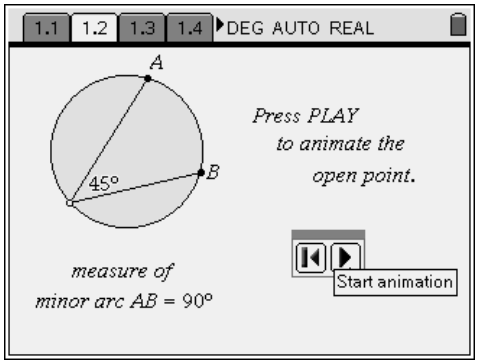
- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet GeoAct27_InscribedAngles_worksheet_EN helps to guide students through the activity and provides a place to record their answers.*
- *The TI-Nspire solution document GeoAct27_InscribedAngles_Soln_EN.tns shows the expected results of working through the activity.*

TI-Nspire™ Applications

Graphs & Geometry, Notes

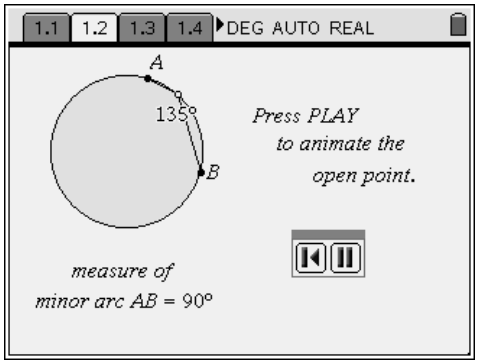
Problem 1 – Discover the rules

Have students move to page 1.2. If needed, explain that the open point is the vertex of an inscribed angle that intercepts $\overset{\frown}{AB}$.



Students can animate the open point by pressing play on the animation control panel. As the point moves around the circle, students will find that when the arc is intercepting the minor arc that measures 90°, the measure of the angle remains 45°.

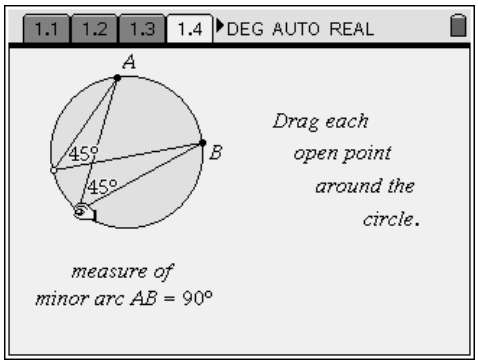
When the vertex moves *between* points *A* and *B*, the measure of the inscribed angle is 135°.



Ask students what the measure of the major arc *AB* is and why (270°; 360° – 90° = 270°).

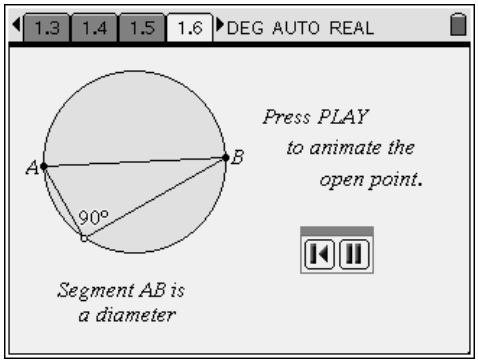
Have students make a conjecture about their observations on page 1.2. They should say that an inscribed angle measures half the measure of the intercepted arc.

On page 1.4, students can simply drag each of the open points around the circle (rather than animating).



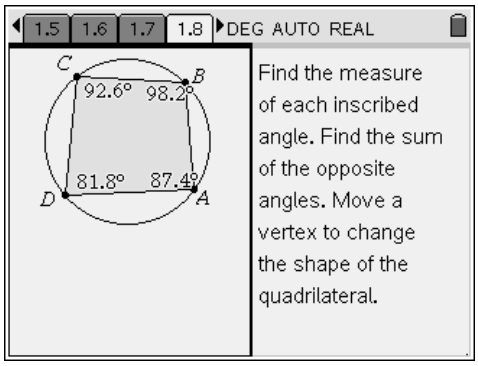
Students should conjecture that inscribed angles that intercept the same arc have equal measures. Explain that this can be extended to say: *Inscribed angles in a circle that intercept the same arc, or congruent arcs, are congruent*.

For the diagram on page 1.6, students can animate the open point as before. They should conclude that if an inscribed angle intercepts a semi-circle, the inscribed angle is a right angle. Ask them to explain why this must be true—a semi-circle measures 180° and the measure of an angle intercepting a 180° arc would be half that, which is 90°.)



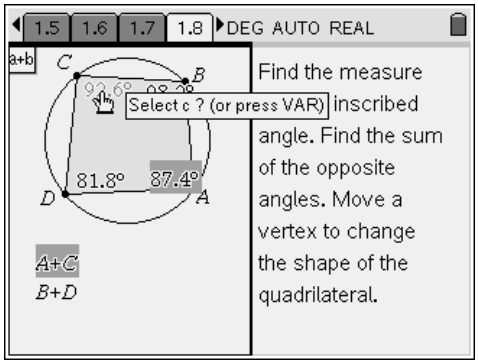
If needed on page 1.8, explain that the quadrilateral is inscribed in the circle because each of its vertices is on the circle.

Tell students to find the measure of each angle by pressing **MENU > Measurement > Angle** and selecting one point on one side of the angle, then its vertex, and then a point on the other side of the angle.

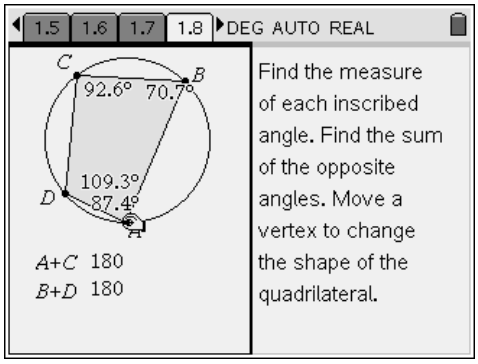


Once students have found the measures of all four angles, instruct them to use the **Text** tool (**MENU > Actions > Text**) to display an expression for the sum of opposite angles, such as *C* + *A* and *D* + *B*.

Next, students should use the **Calculate** tool (**MENU > Actions > Calculate**) to find the sums of the opposite angles. After selecting the **Calculate** tool, click first on the expression to be evaluated, and then on the values for each of the variables as prompted.

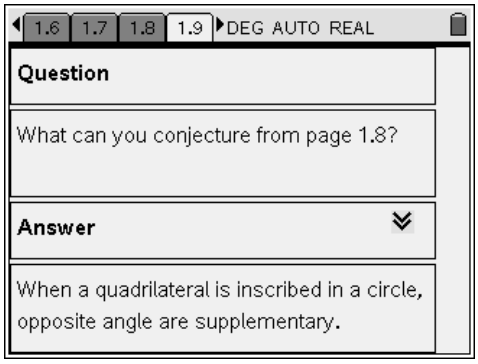


Students will see that the current measures indicate that the opposite angles are supplementary. Tell students to drag any vertex to change the shape of the quadrilateral to see if this remains true.



Have students state their conjecture on page 1.9.

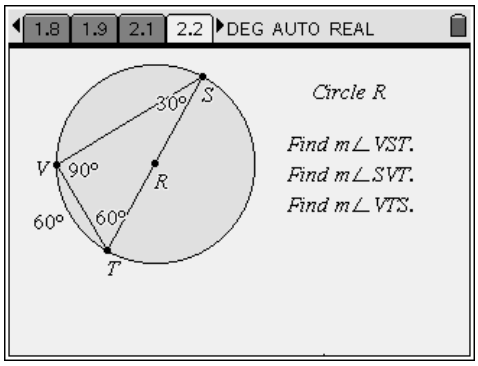
Discuss why it makes sense for the opposite angles to be supplementary. (The two intercepted arcs form a full circle which measure 360°; the sum of the inscribed angles that intercept those arcs is half that, or 180°)



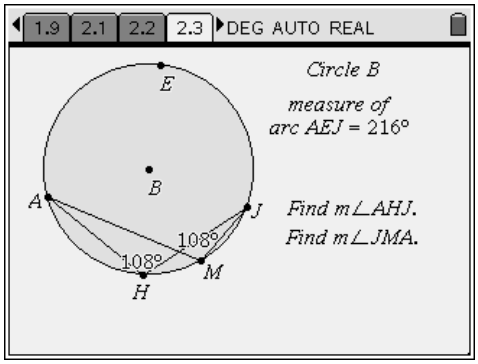
Problem 2 – Use the rules

Have students work though pages 2.2, 2.3, and 2.4 on their own. Discuss the correct answers when everyone has finished. Include the following in your discussion.

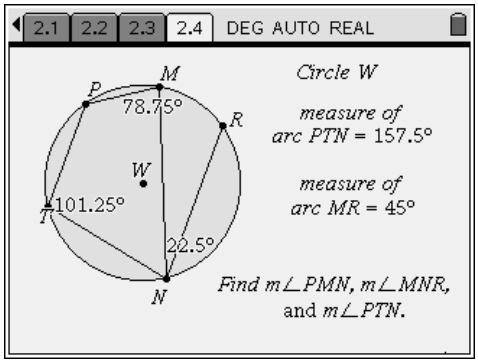
On page 2.2, $m\angle T$ can be found be using either the Triangle Sum Property, or by subtracting 60° ($m\overset{\frown}{VT}$) from semi-circle *TVS* which measures 180° and taking half of that measure.



Extend the problem on page 2.3 by asking students how they can find $m\angle AEJ$. (Subtract 216° from 360° to find $m\overset{\frown}{AJ}$, then divide by 2, $m\overset{\frown}{AJ} = 72^\circ$.) Then have students confirm this by finding the measure on their handheld.



Extend the problem on page 2.4 by asking students to describe two ways to find $m\overset{\frown}{PRN}$. (Double 101.25° because it is the intercepted arc of $\angle PTN$, or subtract 157.5° from 360°.) Tell students that $m\angle TNM$ = 56.25° and ask how they can use this to find $m\overset{\frown}{TNM}$ and $m\overset{\frown}{TPR}$. ($m\overset{\frown}{TPM} = 2m\angle TNM = 112.5^\circ$. Subtract this from 360° to find $m\overset{\frown}{TNM}$, 247.5°. To find $m\overset{\frown}{TPR}$, add the measures of arcs *TPM* and *MR*: 112.5° + 45° = 157.5°. Alternatively, double the sum of $m\angle TNM$ and $m\angle MNR$.)



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(Student)TI-Nspire File: *GeoAct27_InscribedAngles_EN.tns*