Sums of Sequences

ID: 11135

Time Required
60 minutes

Activity Overview

In this activity, students will develop formulas for the sum of arithmetic and geometric sequences. Students will then find the sum of sequences using the formulas developed.

Topic: Sequences, Series, & Functions

- *Arithmetic Sequences*
- *Sum of Arithmetic Sequences*
- *Geometric Sequences*
- *Sum of Geometric Sequences*

Teacher Preparation and Notes

- *Before the activity, the teacher should make sure students are familiar with arithmetic and geometric sequences, including notation for sequences. The TI-Nspire document needs to be downloaded to the student handhelds.*
- *Teacher will need to lead a discussion about student findings in the beginning of each part of the activity.*
- *The sum of a geometric sequence is only for sequences beginning with 1.*
- *Formulas needed:* $\frac{n(a_1+a_n)}{2}$, $\frac{1-r^n}{1-r}$
- *The questions in the student tns file can be assigned for homework if the teacher wishes to focus on the developing formula part of the activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11135" in the quick search box.***

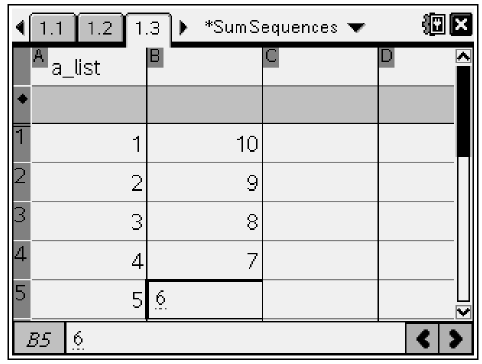
Associated Materials

- *SumSequences_Student.doc*
- *SumSequences.tns*
- *SumSequences_Soln.tns*

Suggested Related Activities

- *Double Tree (TI-Nspire technology) — 9772*
- *Geometric Sequences & Series (TI-Nspire technology) — 8674*
- *Arithmetic Sequences & Series (TI-Nspire technology) — 8638*

Problem 1 – Sum of Arithmetic Sequences



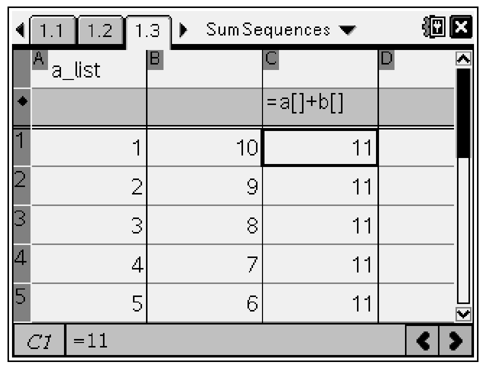
In part 1 of the activity, students will develop the formula for an arithmetic sequence. They are given lists in spreadsheets and will perform a numeric proof to find the formula.

Directions for students on worksheet:

Step 1: On page 1.3, examine the given sequence, called **a_list**.

Step 2: In Column B, enter the sequence in reverse order.

Step 3: In the grey box of Column C, enter the formula **=a + b**.

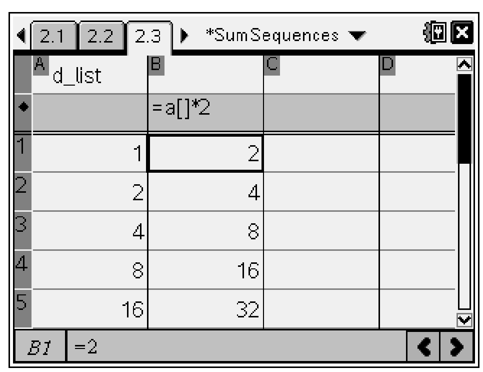


After students complete the first problems, the teacher should lead a discussion to have students determine the formula based on the process they followed. Questions to ask may include:

- How did you find the same sum each time?
- What number did you multiply by each time?
- How is this number related to the sequence?
- Why doesn't this process give us the answer immediately?
- How do we get to our answer?

Students can work on the problems on pages 1.7 to 1.10 in class or for homework.

Problem 2 – Sum of Geometric Sequences



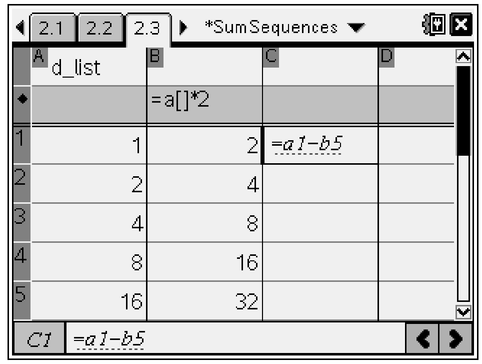
In this part of the activity, students will determine the formula for the sum of a geometric sequence. Students are given sequences in a spreadsheet and will follow steps to develop the sum formula.

Directions for students on worksheet:

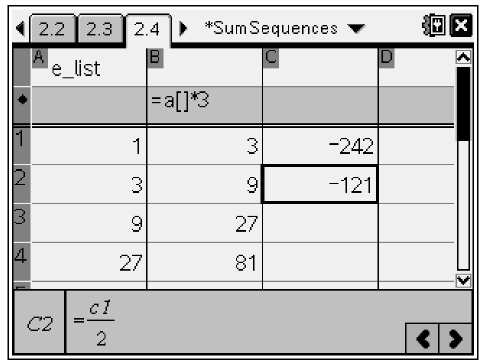
Step 1: On page 2.3, examine Column A to determine the common ratio between the terms.

Step 2: In the grey box of Column B, enter the formula **=a*(Common Ratio)**, where Common Ratio is the number you found in Step 1.

Step 3: Notice the diagonals of the two columns have the same values. There are only two values that are different. In cell C1, subtract the two remaining values by typing in a formula. Note: You will only have one formula in the one cell.



Step 4: Determine what number you need to **divide** the value in cell C1 by in order to get the sum of **b_list**. (Sum the values in Column A mentally.)



After or while students work, the teacher should help students relate the sum of the series, calculated mentally or by inserting a *Calculator* page, to the process they are following in these steps. Remind students they are working on finding a formula to work for every sequence, not just the example sequences given in the activity.

Students will probably need to adjust their conjectured formula after repeating the exercise for the next the sequences on pages 2.4 and 2.5. Make sure students have the correct formula before answering the questions.

Questions to ask may include:

- Why do the diagonals cancel out?
- What role does the common ratio play in having values cancel out?
- How can you write the terms using the common ratio?
- Why does dividing make sense for this formula?
- Compare the process for arithmetic sequences to geometric sequences.

Students can work on the problems on pages 2.7 to 2.9 in class or for homework.

Extension – More Sums of Geometric Sequences

Have students start the sequence with a number other than one. How does this affect the sum?

This should help develop the complete formula and allow the teacher to lead the class in an algebraic proof of the formula.

Solutions – Student Worksheet

Problem 1 – Sum of Arithmetic Sequences

- The sums are the same.
- $11 \times 10 = 110$
- The sum of Column C is twice the sum of Column A.
- The form of the formula given by students may vary.
  $\frac{n(a_1 + a_n)}{2}$
- Page 1.7: $\frac{6(2+12)}{2} = 42$

  Page 1.8: $\frac{8(7+35)}{2} = 168$

  Page 1.9: $\frac{7(9+(-39))}{2} = -105$

  Page 1.10: $\frac{12(3+113)}{2} = 696$

Problem 2 – Sum of Geometric Sequences

- $2^0, 2^1, 2^2, 2^3, 2^4$
- $1 - 2^5$
- The form of the formula given by students may vary.
  $\frac{1-r^n}{1-r}$
- Page 2.7: $\frac{1-\left(\frac{1}{2}\right)^6}{1-\left(\frac{1}{2}\right)} = \frac{63}{32} = 1.96875$

  Page 2.8: $\frac{1-\left(\frac{1}{3}\right)^5}{1-\left(\frac{1}{3}\right)} = \frac{121}{81} \approx 1.49383$

  Page 2.9: $\frac{1-(-2)^7}{1-(-2)} = 43$

