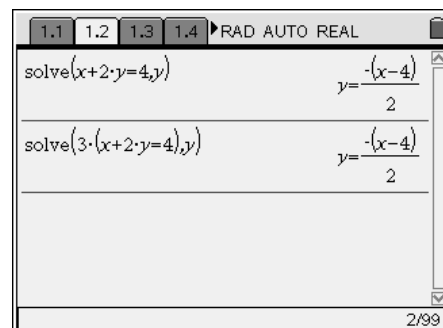


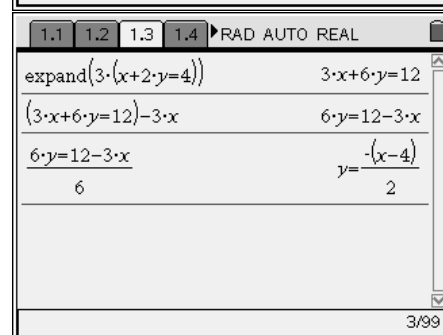
## Solving Systems of Linear Equations with Linear Combinations (The Elimination Method)

### Problem 1—Launch the Activity (Multiplying an Equation by a Constant)

When asked to graph the equations  $x + 2y = 4$  and  $3(x + 2y = 4)$ , a student in Mr. Kennedy’s algebra class used a CAS to solve for  $y$  (as seen on page 1.2).



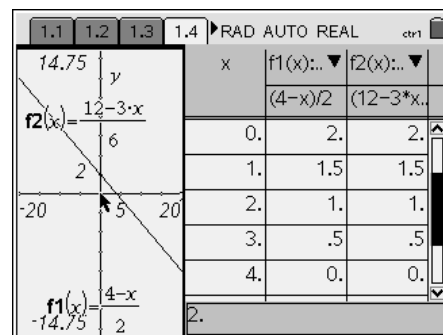
Confused by the CAS results on page 1.2, the student tried an algorithmic method for solving for  $y$  using the CAS (as seen on page 1.3).



Now convinced that the CAS has made a mistake, the student decides to symbolically (without CAS) solve these equations for  $y$ . The student’s work is shown below.

$x + 2y = 4$ $x + 2y + (-x) = 4 + (-x) \quad (\text{add } -x)$ $2y = 4 - x$ $\frac{2y}{2} = \frac{(4 - x)}{2}$ $y = \frac{(4 - x)}{2}$	$3(x + 2y = 4)$ $3x + 6y = 12$ $3x + 6y + (-3x) = 12 + (-3x) \quad (\text{add } -3x)$ $6y = 12 - 3x$ $\frac{6y}{6} = \frac{12 - 3x}{6}$ $y = \frac{12 - 3x}{6}$
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Next the student graphed  $f1 = \frac{4 - x}{2}$  and  $f2 = \frac{12 - 3x}{6}$  on the same coordinate plane and added a function table to compare (see page 1.4).

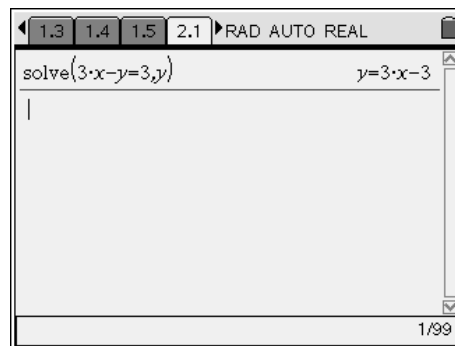


### Question

1. What is true about the equations  $x + 2y = 4$  and  $3(x + 2y = 4)$  that this student seems unwilling to except? Explain.

## Problem 2—Multiplying an Equation by a Constant (Continued)

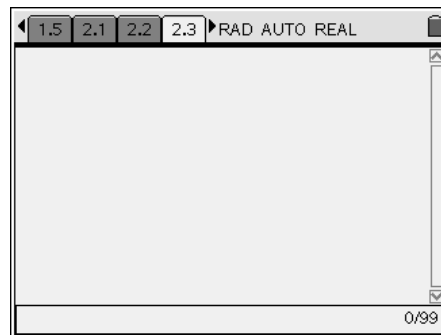
In page 2.1, use CAS to solve  $3x - y = 3$ ,  $2(3x - y = 3)$ ,  $3(3x - y = 3)$  and  $5(3x - y = 3)$  for  $y$ .



### Questions

1. What is true about these equations? How do you know this?
2. What do you think is true about the equations  $3x + 2y = 8$  and  $k \cdot (3x + 2y = 8)$ , where  $k$  is any nonzero real number? Symbolically (without CAS) solve each of these equations for  $y$ . Does this justify your conjecture? Explain.
3. What do you think is true about the equations  $ax + by = c$  and  $k \cdot (ax + by = c)$ , where  $a$ ,  $b$ ,  $c$ , and  $k$  are nonzero real numbers? Explain.

In page 2.3, use CAS to symbolically verify your conjecture in question 3. Explain your work.

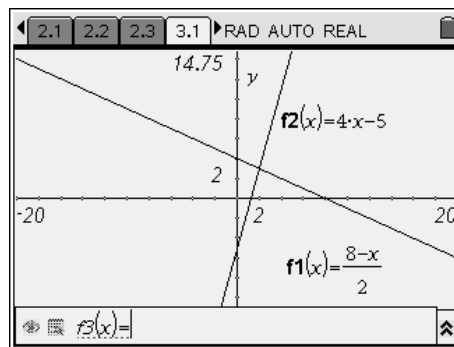


### Problem 3—Adding Equations

In **Problem 2**, you explored the affect of multiplying an equation by a constant and found that (although it may change the appearance) it does not change the equality relationship that existed in the original equation. In this problem, an additional linear equation is introduced to form a system of two linear equations.

When asked to graph  $x + 2y = 8$  and  $4x - y = 5$ , a student in Mrs. Reed's class produced the results found in page 3.1

Note: to graph these equations this student first solved each equation for  $y$  so that they could be graphed as functions.



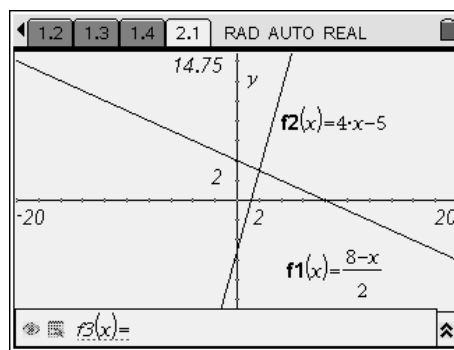
#### Question

1. Does this system of equations have a solution? Explain.

Add  $x + 2y = 8$  and  $4x - y = 5$ , or,

$$\begin{array}{r} x + 2y = 8 \\ 4x - y = 5 \\ \hline 5x + y = 13 \end{array}$$

Graph the sum equation ( $5x + y = 13$ ) as **f3** in page 2.1



#### Question

1. How is the graph of the sum equation related to solution of the original system of equations?

Multiply  $x + 2y = 8$  by  $-4$ .

#### Question

1. How is this new equation related to  $x + 2y = 8$ ?

Add  $4x - y = 5$  and the equation you found when you multiplied  $x + 2y = 8$  by  $-4$ .

### Questions

1. What is special about this sum equation?
2. Make a conjecture about the relationship that the graph of this sum equation has to the original system of two equations. Check your conjecture by graphing the sum equation as **f4** in page 3.1. Explain your findings.
3. If  $E_1$  and  $E_2$  are linear equations intersecting at point  $(a, b)$ . What can you say about the **linear combination**  $m \cdot E_1 + n \cdot E_2$ , when  $m$  and  $n$  are nonzero numbers?

### Problem 4 – The Elimination Method

Consider a different system of two linear equations:  $5x + 2y = 4$   
 $x + 3y = -7$

In the work below, a student uses linear combinations (sometimes called the **elimination method**) to solve this system of equations. Explain what the student is doing in each step and why it is possible to do this.

Student Solution	Explanation (Why)
(1) $-5 \cdot (x + 3y = -7)$ $-5x - 15y = 35$	
(2) $5x + 2y = 4$ $\underline{-5x - 15y = 35}$ $-13y = 39$	
(3) $\frac{-13y}{-13} = \frac{39}{-13}$ $y = -3$	
(4) $5x + 2(-3) = 4$ $5x - 6 = 4$ $5x - 6 + 6 = 4 + 6$ $5x = 10$	
(5) $\frac{5x}{5} = \frac{10}{5}$ $x = 2$	

In the previous exercise, the student solved a system of linear equations using the elimination method. Now it is your turn.

Without the use of technology, solve each of the following systems of two linear equations with the elimination method. Check your solution by solving the system using another method. Explain your processes.

a.  $x + 2y = 4$   
 $x - y = 5$

b.  $2x + 3y = 6$   
 $2x + y = -4$

c.  $x + 3y = -1$   
 $3x - y = 12$

d.  $5x + 4y = -2$   
 $-2x - 3y = 1$

### Problem 5—A Special Case

Consider this system of equations. Solve this system of linear equations using the elimination method. Check your solution with another method.

$$\begin{aligned}2x - y &= 6 \\ -4x + 2y &= 8\end{aligned}$$

#### Questions

1. What did you notice?
  
2. What does this mean about the system? How do you know this?

**WARNING:** At the end of the activity, do not save the changes. Close the document and answer NO to saving changes. You may want to be able to open up the file again without all the data that you gathered and graphed.