NUMB3RS Activity: To Pythagoras and Beyond
Episode: “Traffic”

Topic: Extending the Pythagorean Theorem **Grade Level:** 9 - 12
Objective: Explore using the Pythagorean relationship with non-right triangles to relate the Pythagorean Theorem to the Law of Cosines.
Materials: ruler, protractor, and a calculator
Time: 15 - 20 minutes

Introduction

In “Traffic,” the FBI enlists Charlie's help after a series of seemingly random highway attacks grips Los Angeles. Larry is working at the chalkboard on some complicated looking mathematics and physics when Charlie comes in and sees that Larry is using the Pythagorean Theorem and the Law of Cosines.

Discuss with Students

This activity is ideal for students working in groups. Each group needs to have three rulers, a protractor, and a calculator.

Remind students that a triangle is classified as acute, right, or obtuse depending on the size of its angles. Review the Pythagorean Theorem and its converse. Indicate that while the theorem is only true for right triangles, there might be a potential for using it with non-right triangles.

Student Page Answers:
1. *90^o* **2.** *They are equal.* **3.** *about 103^o* **4.** *$6^2 + 8^2 < 11^2$ The angle is obtuse.* **5.** *The angle is obtuse in the first two triangles and acute in the third.* **6a.** *acute* **6b.** *obtuse* **6c.** *right* **6d.** *obtuse* **7.** *No; you should compare $10^2 + 6^2$ with 15^2 to conclude that the triangle is obtuse.* **8.** *The Law of Cosines becomes the Pythagorean Theorem in the case the triangle is right, since cos 90° = 0.* **9.** *$c^2 < a^2 + b^2$ implies that cos C > 0 and therefore that $\angle C$ is acute.* **Extension:** *The essence of the proof is as follows: Through the use of transformations, areas of pairs of rectangles are shown to be equal. These pairs are (A1, A2), (A3, A4), and (A5, A6). The area of both rectangles A5 and A6 is shown to be bc cos A. Thus, $a^2 = \alpha A2 + \alpha A3 = \alpha A1 + \alpha A4 =$ $(c^2 - \alpha A6) + (b^2 - \alpha A5) = c^2 + b^2 - 2bc \cos A$, where $\alpha A1$ represents the area of rectangle A1, etc.*

John F. Mahoney, Benjamin Banneker Academic High School, Washington, DC

Name: ________________________________ Date: _______________

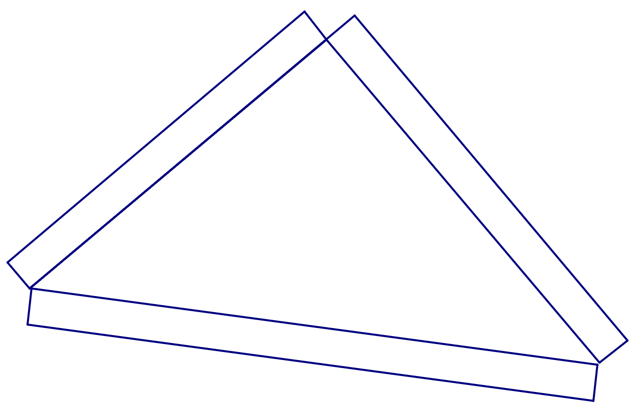
NUMB3RS Activity: To Pythagoras and Beyond
Episode: "Traffic"

In "Traffic," the FBI enlists Charlie's help after a series of seemingly random highway attacks grips Los Angeles. Larry is working at the chalkboard on string theory physics when Charlie comes in and sees that Larry is also using the Pythagorean Theorem and the Law of Cosines.

The Pythagorean Theorem is essential to much of advanced physics and mathematics. The theorem states that in a right triangle, the sum of the squares of the lengths of two legs is equal to the square of the length of the hypotenuse. Algebraically, if a and b are the lengths of the legs and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$. However, suppose you tried to apply the Pythagorean Theorem to a non-right triangle? What other relationships exist?

This activity requires three rulers and a protractor. You can use either centimeters or inches, but be consistent. In this activity, you will work with triangles that have two sides whose lengths are 6 and 8.

1. Lay the three rulers out so that a triangle is formed whose sides are 6, 8, and 10.



Using your protractor, find the measure of the angle between the sides of lengths 6 and 8. __________

2. What is the relationship between the two quantities $6^2 + 8^2$ and 10^2? __________

3. Rearrange the rulers so they form a triangle that has sides of lengths 6, 8, and 11. What is the measure of the angle between the sides of lengths 6 and 8?

John F. Mahoney, Benjamin Banneker Academic High School, Washington, DC

4. What is the relationship between the two quantities: $6^2 + 8^2$ and 11^2? Is the angle between the sides of lengths 6 and 8 acute, right, or obtuse?

5. Repeat Questions 3 and 4 with triangles with side lengths 6, 8, 12; 6, 8, 13; and 6, 8, 9. Try some other side lengths and fill in the chart below. What is the relationship between the type of the angle formed by the sides of lengths a and b and which of $a^2 + b^2$ and c^2 is greater?

a	***b***	***c***	$a^2 + b^2$	c^2	**Is the angle right, acute, or obtuse?**
6	8	12			
6	8	13			
6	8	9			

6. The relationship that you discovered in question 6 can be used to test whether the triangle is acute, right, or obtuse. Each list below gives the side lengths of a triangle. Determine whether each triangle is acute, right, or obtuse.

a) 8, 12, 13 **b)** 10, 6, 15 **c)** 28, 45, 53 **d)** 38, 80, 89

7. Look at the statement below.

A triangle has sides 10, 6, and 15. Because $15^2 + 6^2 > 10^2$*, the triangle is acute.*

Is this statement true? Why or why not?

The Law of Cosines states that in any triangle with sides a, b, and c,
$c^2 = a^2 + b^2 - 2ab \cos C$, where C is the angle between sides a and b.

8. How does Law of Cosines become simpler when angle C is a right angle?

9. Show how the Law of Cosines can be used to explain the observation from Question 7 that $c^2 < a^2 + b^2$ implies that the angle is acute.

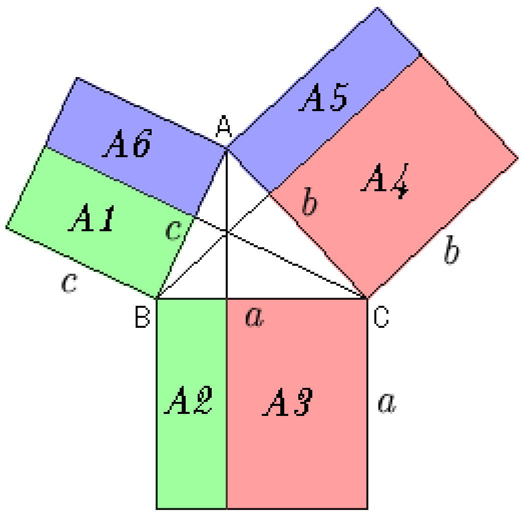
John F. Mahoney, Benjamin Banneker Academic High School, Washington, DC

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

Visit the Web site **http://www.ies.co.jp/math/java/trig/yogen_auto/yogen_auto.html**. This Web site has a Java applet that illustrates a proof of the Law of Cosines based on squares on the sides of a triangle. A diagram from the applet is shown below:



[Source: **http://www.ies.co.jp/math/java/trig/yogen_auto/yogen_auto.html**]

Use the applet to help you write a detailed proof of the Law of Cosines. How does this proof compare with the one in your textbook?

Additional Resources

- This Web site below has an applet that can be used to explore the answer to the question "what happens to the Pythagorean Theorem for acute triangles or obtuse triangles?" **http://www.keymath.com/DG/dynamic/law_of_cosines.html**
- For a generalization of the Pythagorean Theorem and geometric interpretations of the Law of Cosines, see **http://ctap295.ctaponline.org/~pgerrode**.

John F. Mahoney, Benjamin Banneker Academic High School, Washington, DC