Teacher Notes

Activity 2

Is There a Limit to Which Side You Can Take?

Objectives

- Given a function, state and explain the limit at a particular value
- Given a graph, state and explain the limit at a particular value

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 40 minutes

Abstract

This activity will examine one-sided limits.

Management Tips and Hints

Prerequisites

Students should:

- have knowledge of piecewise functions.
- be able to produce piecewise functions on the graphing handheld.
- be able to manipulate graphs and tables of values manually and use the [ZOOM] features of the graphing handheld.
- have a basic understanding of function language and rational, exponential, and trigonometric functions.

Student Engagement

Working in pairs or small groups is recommended for maximum student engagement.

Evidence of Learning

- Given a function, students will be able to state and explain the limit at a particular value using graphical and numerical analysis.

Common Student Errors/Misconceptions

- Students may not produce piecewise functions correctly.
- Students may misunderstand the behavior of a piecewise function.
- Students may incorrectly interpret the notation $\lim_{x \to 3^+}$ to mean moving toward the value (here 3) as it gets larger.
- Students may incorrectly interpret the notation $\lim_{x \to 4^-}$ to mean moving toward the value (here 4) as it gets smaller.
- Students may misinterpret the value of the limit of a function with the value of a function at a point.
- Care should be taken to make sure that students understand the difference between an actual graphing handheld error and the fact that the function is not defined at a particular point.

Activity Solutions

1. n/a

2. n/a

3. $f(x) = \{^-2, \text{ERROR}, 4\}$

4.

X	Y_1	Y_2
1.7	-1.3	ERROR
1.8	-1.2	ERROR
1.9	-1.1	ERROR
2	ERROR	ERROR
2.1	ERROR	3.1
2.2	ERROR	3.2
2.3	ERROR	3.3

X=2

5.

X	Y_1	Y_2
1.97	-1.03	ERROR
1.98	-1.02	ERROR
1.99	-1.01	ERROR
2	ERROR	ERROR
2.01	ERROR	3.01
2.02	ERROR	3.02
2.03	ERROR	3.03

X=2

6. There is no limit as x approaches 2. As the input value gets nearer and nearer to 2, the function value never gets near any one value.

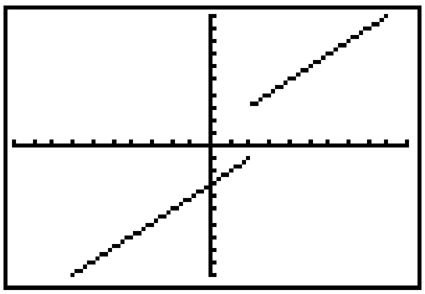
7. $\lim_{x \to 2^+} f(x) = 3$

As the input moves toward 2 from the right side, the function continues to get closer to a value of 3.

8. $\lim\limits_{x \to 2^-} f(x)$

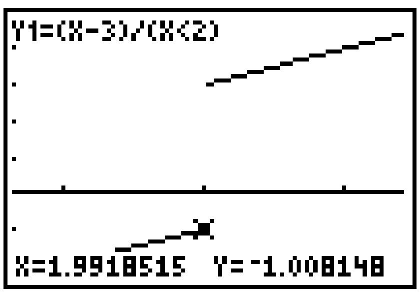
means finding the limit of the function as the input value is approached through values that are less than 2 (left side). The "useful" input values will vary, but 1.97, 1.98, 1.99 would make sense because they were used in the previous table. The limit from that side is $^-1$.

9.

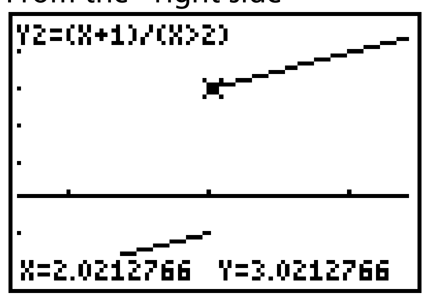


10. Possible graphs:

From the "left side"

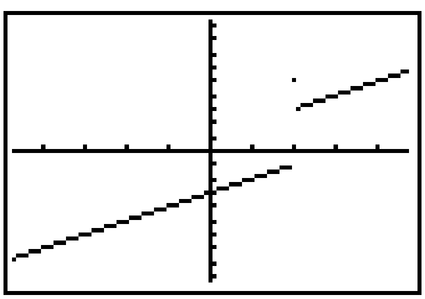


From the "right side"



Answers will vary. The goal is to have the student see the values approach the "one-sided" limits concluded in the numerical analysis.

11.

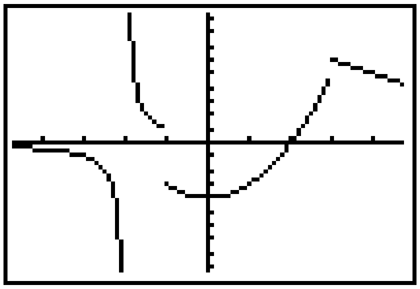


12. The only difference is that the function *g*(*x*) is defined at *x* = 2, and the function *f*(*x*) is not.

13. $\lim\limits_{x \to 2} g(x)$ = no limit $\lim\limits_{x \to 2^+} g(x) = 3$ and $\lim\limits_{x \to 2^-} g(x) = {}^-1$.

Some students may say that at 2 there is a limit of 5 because the function is defined at that point. This is the opportunity to firm up the concept of limit as the behavior a function has as the input gets *very near* a particular value.

14.



15.

a. No limit; there is a vertical asymptote at ⁻2.

b. $+\infty$; the function will continue in the positive direction.

c. No limit; there are two different one-sided limits.

d. 1; explanations will vary, but some students will trace, and some will use numerical conclusions.

e. No limit; two different one-sided limits.

f. 6; explanations will vary, but some students will trace, and some will use numerical conclusions.

16.

a. 6

b. ⁻2

c. ⁻2

d. 0

e. ⁻2