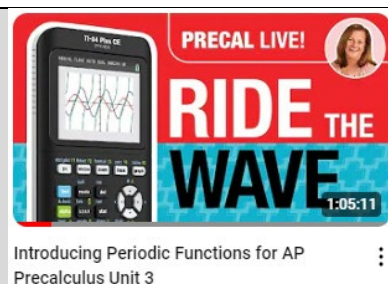





Thursday Night Precalculus Series

January 11, 2024

In this AP Precalculus Live session, we will explore periodic functions and their characteristics.



About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand:
 - <https://www.youtube.com/live/hizywSt0N8g?si=qF1r5peeX3EQ2Aiu>
 - Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the “Live Session” but can be enriched by that resource.*
- This session involves exploring features of the graphs of periodic functions and their characteristics, such as:
 - Finding the period.
 - Determining intervals of increase and decrease.
 - Determining concavity.
 - Determining rates of change.
- Students should be able to use the TI-84 to verify these features of a periodic function.
-  **Class Discussion:** Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

Materials:

Student document

- PreCal_problems_01_11Solutions
- PRECAL_problems_solutions_01_11

YouTube

- <https://www.youtube.com/live/hizywSt0N8g?si=qF1r5peeX3EQ2Aiu>

AP Precalculus Learning Objectives

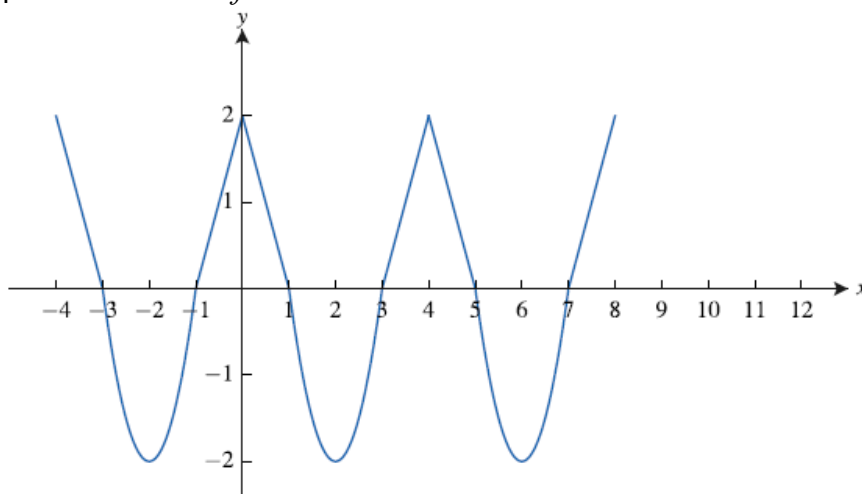
- 3.1.A: Construct graphs of periodic relationships based on verbal representations
- 3.1.B: Describe key characteristics of a periodic function based on a verbal representation.

Source: AP Precalculus Course and Exam Description, The College Board



Problem 1. (a) – (c)

The graph of a periodic function f is shown.



- (a) What is the period, p , of the function?
- (b) Sketch the next period of the given graph.
- (c) Determine whether each function is periodic. If it is, state the period. If it is not, explain why.
- (i) $y = f\left(\frac{1}{2}(x-1)\right)$
- (ii) $y = -f(x)$
- (iii) $y = f(-x)$
- (iv) $y = f(2x)$
- (v) $y = f(x^2)$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.



Class Discussion:

What does it mean for a function to be periodic? How do we find the period?

Possible Answers: A function is periodic if there exists a value for p so that

$f(x+p) = f(x)$. Visually there is a “chunk” of the graph that gets repeated every p units.

When we look at a graph, we are trying to determine that part of the graph that gets repeated.

**Class Discussion:**

How can we use the transformations (additive and multiplicative) from Units 1 and 2 to explain whether or not each function in (c) is also periodic?

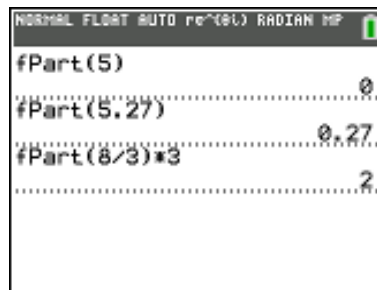
Possible Answers: Using (c) (i), the $(x-1)$ is a horizontal translation to the right 1 unit and the $\frac{1}{2}$ is a horizontal dilation by a factor of 2, so the resulting graph is periodic. A sketch of the new function is also very helpful.

Teacher Note: Spend time on Topic 3.1 to emphasis vocabulary and the transformations from Unit 1 and 2 with “nontraditional” periodic functions.

Demonstration of the generation of a periodic function on the TI-84.

The demonstration of generating a periodic function using fPart(is beneficial to the teacher in writing or creating questions and documents to use in class. The demonstration of using the TI-84 follows the demonstration on the TI-Nspire.

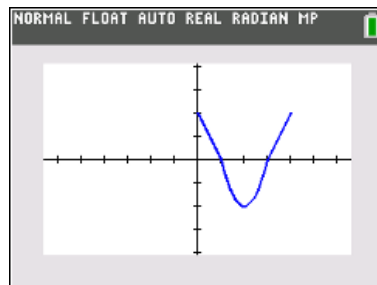
Technology Tip: To access fPart(select Math > Num > 4: fPart(. This operation “lops” off the whole number part and leaves the fraction.

**Graphing a Piecewise Function on the TI-84**

Here is a Youtube Video in the Texas Instruments Education Channel on graphing piecewise functions.

https://youtu.be/lfvMHcC3tVo?si=CaHLilpiSg6yQ_UM

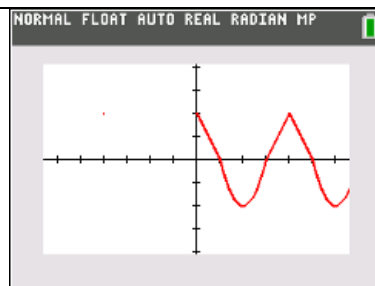
$$\text{Graph } Y_1(x) = \begin{cases} 2 - 2x; & 0 \leq x \leq 1 \\ 2(x-1)(x-3); & 1 < x < 3. \\ 2(x-3); & 3 \leq x \leq 4 \end{cases}$$





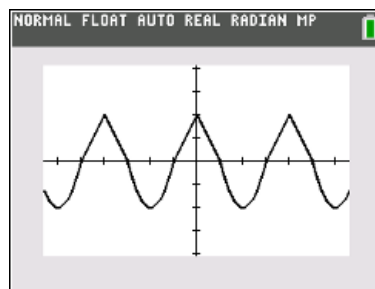
We want to repeat the piece of the function from $x = 0$ to $x = 4$ to create a periodic function.

Graph $Y2(x) = Y1(\text{fPart}(x/4)*4)$.

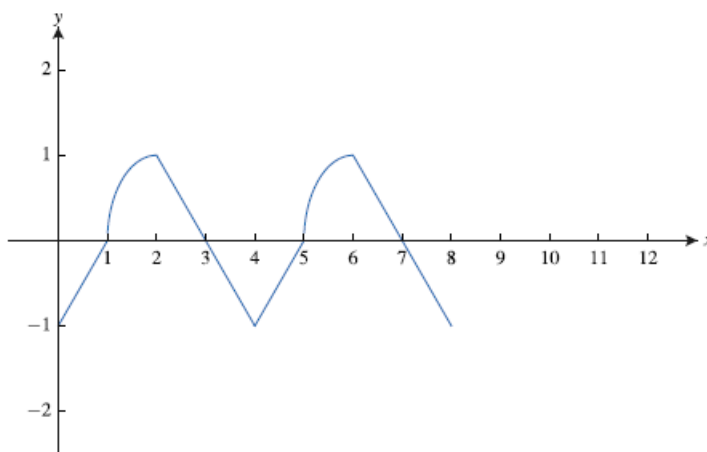


To shift the graph a couple of periods over to the left, graph

$Y3(x) = Y2(x+8)$.

**Problem 2.**

The graph of a periodic function f is shown below.



- (a) Sketch another cycle of the function on the interval $[8, 12]$.
- (b) Find $f(14)$ and $f(-1)$.
- (c) Find the open intervals for $0 \leq x \leq 8$ on which the function is increasing and concave down.
- (d) Find the open intervals for $0 \leq x \leq 8$ on which the function is decreasing and concave up.

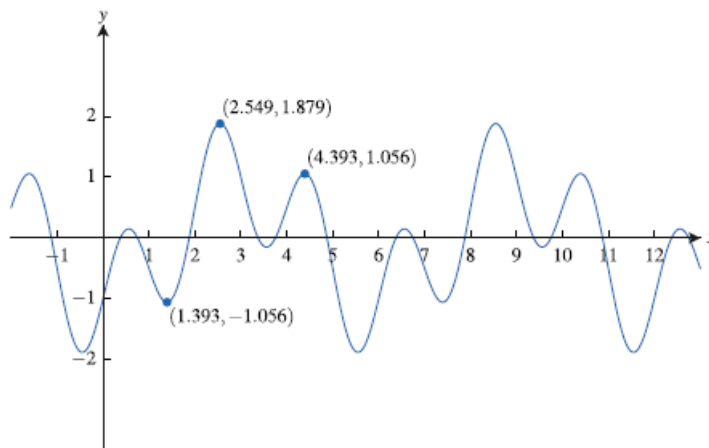
Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.



Problem 3.

The graph of a periodic function f is shown below.



- (a) Write an expression for a function g that is a horizontal translation of the graph of f which would be the exact same graph as that of f .
- (b) Using the period of f , find the number of complete cycles of the graph of f in the xy -plane on the interval $0 \leq x \leq 350$.

Sample Solution:

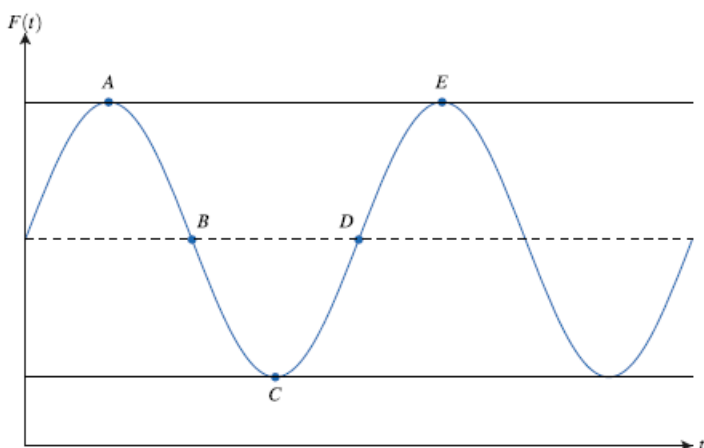
Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 4.

The blades of a large industrial fan rotate in a clockwise direction and rotate at a rate of 10 revolutions per second. Let point A be at the tip of the blade that is straight up at time $t = 0$. Point A is 75 inches from the floor. Each blade has a length of 14 inches from the center.

Let the periodic function F model the distance between point A and the floor, in inches, as a function of time t in seconds.

- (a) Use the given information to find possible coordinates $(t, F(t))$ of the points A , B , C , D , and E on the graph below.



- (b) Use the graph of $y = F(t)$ and the intervals from A to B, B to C, etc. to find an interval on which the graph of F is increasing and concave down.
- (c) Find an interval on which the graph of F is decreasing and concave down.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The graphing application can be used to explore periodic functions.
- The graphing application can be used to explore the behavior of a periodic function.

For more videos from the AP Precalculus Live series, visit our playlist

https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI

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