NUMB3RS Activity: Creating "Random" Numbers
Episode: "Bettor or Worse"

Topic: Pseudo-random Numbers   **Grade Level**: 9-12
Objective: Introduce pseudo-random numbers
Time: about 15 minutes

Introduction

A number is random if it is selected by chance from a group of numbers and if each selection is an independent event. Often, however, a function is used to create random numbers from an initial starting point, a *seed*, and then each successive result is used as the seed to find the next result. This can be written as a recursive function where $x_1 = f(x_0)$, $x_2 = f(x_1)$, and so on, but requires an initial value, x_0, the seed. In this case, the selection of the next number is not independent and the results are not truly random. These numbers are known as *pseudo-random*.

Discuss with Students

Random numbers are used every day in many different ways. Banks use random numbers to make codes to send information over the internet; lotteries use random numbers to choose winners; computer games use random numbers to determine what will happen next. Depending on their use, it may be important or not important that the random numbers are truly random.

In cryptology, it is important that the numbers are truly random. If they are predictable in any way, computers can be used to break the code and steal information. In other uses, it is only important that the numbers appear random to a casual observer. Consider a CD player with a "random" mode. The important thing for the CD player is that the tracks are played in an order that appears random, whether they truly are random or not.

Example When one needs to find random numbers, many simple techniques are available:
- Suppose we needed to choose, very quickly, between two options – 1 and 2. What is an easy way to make this choice randomly? [Example: *flip a coin*]
- Suppose we needed to choose, very quickly, among six options – 1 through 6. What is an easy way to make this choice randomly? [Example: *roll a fair number cube*]
- Suppose we needed to make a choice, very quickly, among one million options – 1 through 1,000,000. What is an easy way to make this choice randomly?

The last situation above shows that sometimes a more sophisticated method is necessary. Often, we will use a calculator or computer to generate large random numbers. But are the numbers that a computer and calculator give us truly random?

In **"Bettor or Worse,"** Charlie explains that when computers or calculators find "random" numbers, the numbers are not *truly* random. The machine is calculating the numbers based on a rule it has been programmed with. Because the rule is complicated and uses very large numbers, the next number is difficult to predict and so is often considered random.

Student Page Answers: **1**: **a** and **b**: *answers will vary* **2:** *Yes (if 'No', ask why)* **3:** *because you need a new seed each time you use it* **4:** *Yes; if they were born on the same numbered day.* **5:** *No; if you get a repeated number and use it as a seed again, all of the first 18 numbers will repeat over and over.*

Name: _____   Date: _____

NUMB3RS Activity: Creating "Random" Numbers

Agent Don Eppes is in the field during an investigation. He needs to assign random two-digit pass codes to 40 members of his FBI task force; the numbers are to be randomly selected from 00 - 99. Don remembers his brother telling him how random number generators work and he quickly makes up a rule to generate pseudo-random numbers. The use of this rule requires a "seed"—a starting number. Since he developed the rule with numbers he made up, he is not sure if it will work. The rule he came up with is:

$$\frac{(seed \bullet 41 + 35)}{101}, \text{ then take the first two decimal places as the result}.$$

Example: Seed is 29: $\dfrac{(29 \bullet 41 + 35)}{101} = \dfrac{(1189 + 35)}{101} = \dfrac{1224}{101} = 12.1188...$

The first two decimal places are the result: **11**.

1. **a**. Use the date (the number of the day) of your birth as the seed for the rule above and find the result using your calculator. _____

   b. Now use the number you found in **1a** as the seed for the rule and find the second value. Repeat, using the second value to find the third, and so on, to find the third through fifth values: _____ _____ _____ _____

2. Does the string of numbers you wrote down appear to be random? _____

3. Using the previous value as the new seed is called *recursion*. Why is recursion helpful in using this random number generator? _____
_____

4. Is it possible that someone else in your class could have the same string of numbers? Explain your answer. _____
_____

5. After using the rule 18 times, Don gets as a result a number he has found before. Can Don ignore that one result and keep making random numbers? Explain your answer. _____
_____.

Extensions

Activity: Details of Pseudo-Random Number Generators

Introduction
Despite overuse of the word 'random', very few events are truly random. If you know enough about the causes of an event (like the rule behind the pseudo-random numbers), then patterns can be seen and the randomness becomes ordered.

For the Student
- Use the rule from the student page: **the first two decimal places of (seed • 41 + 35)/101**. Write down the resultant two-digit numbers when you use each of the numbers 1 – 10 as your seeds. Only use the rule once per seed.
  - Look for a pattern in the numbers you found. How does this pattern relate to the original rule?
- Use your calculator to write a simple program to generate random numbers. Make a rule like the one used above.
  - Some numbers are better than others as parts of the rule. For example, what numbers would make poor divisors?
  - Most random number generators use the function ***modulus*** (mod) to control the output. Find out what this function does and use it in your random number generator.
- Your calculator is likely already programmed with a pseudo-random number generator. The following instructions are written for the TI-84 family of calculators.
  - Use the **rand** function on your calculator to make lists of random numbers.
  - To "seed" the random number generator, input **15 STO> rand ENTER**. Using the **rand** function now will start the sequence with the seed of 15. By reseeding the function, you can recreate the same "random" sequence.
  - Note that the **rand** function on your calculator returns numbers between 0 and 1. What could you do to the result if you wanted to return a number between 0 and 100? Write a command to do this.

Additional Resources
http://www.cut-the-knot.org/probability.shtml
This page includes a simple pseudo-random number generator with an explanation of how it works. The rest of the website is a trove of interesting mathematical puzzles and ideas.

More on Pseudo-Random Numbers

- Find out what other everyday objects or systems use pseudo-random numbers in their operations.
- Mathematician Robert Coveyou said "The generation of random numbers is too important to be left to chance." Explore the meaning of this statement by researching the importance of truly random numbers in cryptography.
- The study of chaos explores underlying rules in otherwise seemingly random phenomena. Research chaos theory and the nature of "truly random".