

NUMB3RS Activity: How Reliable is the Test? Episode: "Calculated Risk"

Topic: Conditional Probability Trees

Grade Level: 9-12

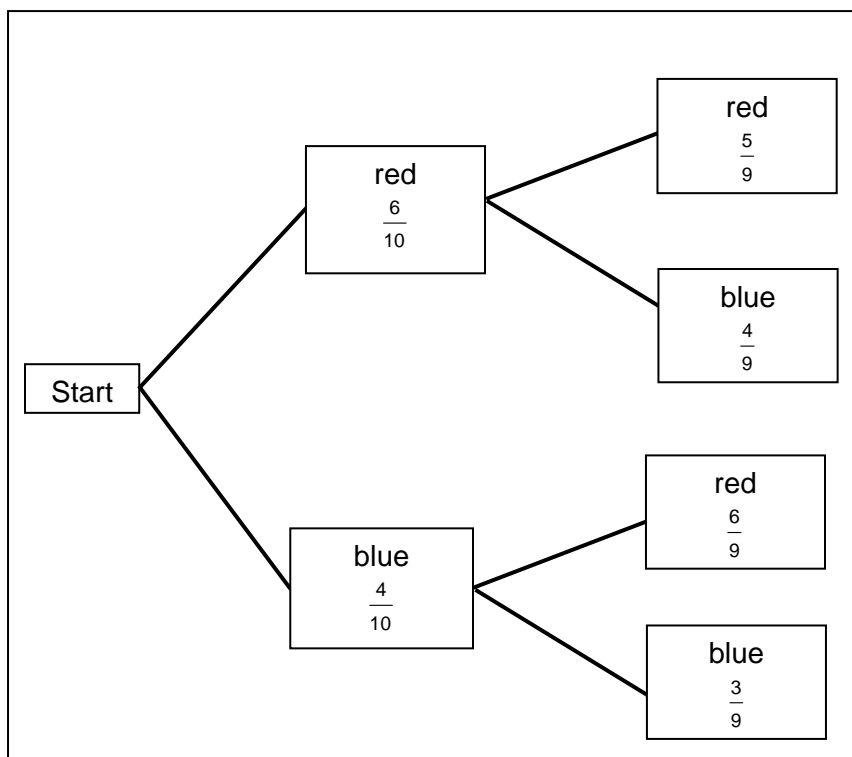
Objective: Understand the construction, meaning, and interpretation of a conditional probability tree

Time: 10-15 minutes

Introduction

Students are familiar with finding the probability of a simple event, such as finding the probability of picking a red gumball from a jar of different colored gumballs. For example, suppose a jar contains 6 red and 4 blue gumballs. The probability of selecting a red one is $\frac{6}{10}$. If a student is asked the probability of selecting a second red gumball, the answer will depend on whether or not the first one has been replaced. If the first one is replaced in the jar, then the probability is again $\frac{6}{10}$ because these two events are *independent*. If the first gumball is not replaced, then the probability becomes $\frac{5}{9}$ and the two events are *dependent*.

Generally, conditional probability (*the chances of A given B*) involves *dependent* events, where the probability of a second event depends on the result of the first event. A tree diagram can help students with the concept of dependent events.



This tree also helps students explore the multiplication theorem for conditional probability, which is $P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$.

So, the probability that the first gumball is red and the second gumball is blue is equal to $\left(\frac{6}{10}\right)\left(\frac{4}{9}\right) = \frac{4}{15}$.

Discuss with Students

In "**Calculated Risk**," Charlie attempts to reduce a cumbersome set of variables to a common point by "pruning" an enormous tree diagram. This requires very complex advanced algorithms, but the basic concept can be appreciated through the understanding of dependent events and conditional probability.

One common use of conditional probability is in the medical industry with drug testing. For example, the probability that a drug will be effective depends on many factors including the health of the person or whether the drug has been administered correctly. Companies try to control these variables, when testing. However, no test is error-proof. From a study, a company can advertise that, for example, drug A has been shown to give a correct result 97% of the time.

In this activity, students will be exploring what the results of a drug test can show. Results such as a false positive (a test showing a positive result when it should have been a negative result) and a false negative (a test showing a negative result which should have been positive) will be discussed along with true positives and true negatives.

Student Page Answers: 1. 0.03395 2. 0.97 3. 0.03 4. $(0.035)(0.97) + (0.965)(0.03) = 0.0629$
 5. $((0.035)(0.97)/(0.0629)) \approx 0.54$ 6. See student work. The four event results are $P(\text{User and Positive})$: 0.03325, $P(\text{User and Negative})$: 0.00175, $P(\text{Non-User and Positive})$: 0.04825, $P(\text{Non-User and Negative})$: 0.91675; the probability that someone who tests positive is a User is ≈ 0.41 , a considerable decrease.

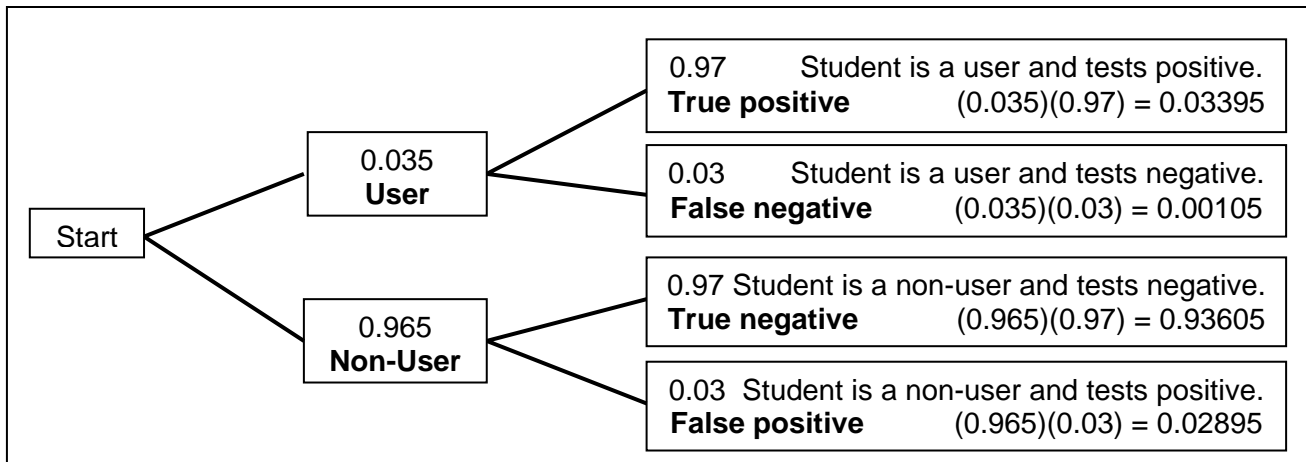
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NUMB3RS Activity: How Reliable is the Test?

Recent national studies indicate that approximately 3.5% of high school students use steroids and related performance-enhancing drugs. The accuracy of the test for the drugs is about 97%. That means that 3% of the time, the test returns an incorrect result (either a false positive or false negative).

This problem can be modeled with a conditional probability tree.



Use the tree to find the probability:

1. of a randomly selected student being both a user and testing positive.

2. of a student testing positive, given that he or she is a user.

3. that a student tests positive, given that he or she is not a user.

4. of a randomly selected student testing positive.

5. of a randomly selected person who tests positive is a user. (Hint: think of this as the ratio of the probability to being a user and testing positive, to the probability of *any* student testing positive.)

6. Construct a tree like the one shown for a new test that has an accuracy of only 95%. Find the probabilities associated with each branch and answer question 4 for your new tree. How do your two answers compare to each other?

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Trees are a way of visualizing the concept of conditional probability. Tables are another way to visualize the data.

A way of determining the probabilities for each is to express some data in the form of a table, as below. This "experimental" data will produce the same tree used on the previous page.

	User	Non-User	Total
Pos.	34	29	63
Neg.	1	936	937
Total	35	965	1000

Example: $35/1000$ (0.035) students are users.

$936/965$ (0.97) non-users tested negative.

For the Student

- Research other tests (medical and otherwise) and their respective success rates. Calculate the probabilities of returning false positive and false negatives. What can testers do to insure that the results returned are as accurate as possible?

Additional Resources

www.enm.bris.ac.uk/teaching/enmdjb/Probability&Statistics/Sec3_Conditional_Prob.pdf

A formal lesson on conditional probability, along with the appropriate notation and a variety of interesting examples.

Related Topic:

These trees are used in decision-making, again using probabilities. Not surprising, these are called "decision trees." Gerber Products Co. (makers of baby products) used a decision tree in order to decide if they should continue to use PVC plastic in their products. A very informative article on this subject is available from Pepperdine University at: <http://gbr.pepperdine.edu/993/tree.html>