EXPLORATIONS™

Activity 19

Objectives

- To investigate the properties of the four classic centers of a triangle

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Centers of a Triangle

Introduction

The centroid, circumcenter, orthocenter, and incenter are the four classic centers of a triangle that are studied in geometry. In this activity, you will construct these center points and discover their properties.

This activity makes use of the following definitions:

Centroid — the point at which the medians of a triangle intersect.

Circumcenter — the point at which the perpendicular bisectors of a triangle intersect.

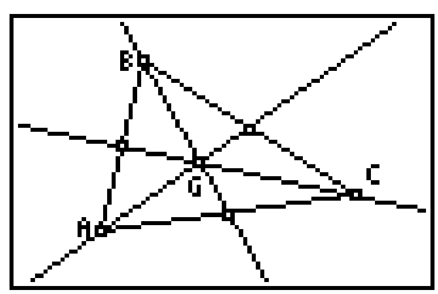
Orthocenter — the point at which the altitudes of a triangle intersect.

Incenter — the point at which the angle bisectors of a triangle intersect.

Part I: Centroid of a Triangle

Construction

Construct a triangle and its centroid.



- Draw $\triangle ABC$ in the center of the screen.
- Construct the three medians of $\triangle ABC$. (See Activity 18 for instructions on constructing a median.)
- Construct a point G where the medians intersect.

Exploration

Use various tools (for example, measurement and circle tools) to investigate the properties of the centroid.

Drag the vertices and sides of the triangle and observe which properties of a centroid are true for any triangle and which are true only for certain types of triangles.

Questions and Conjectures

Describe at least two properties of a centroid that you observed.

Part II: Circumcenter of a Triangle

Construction

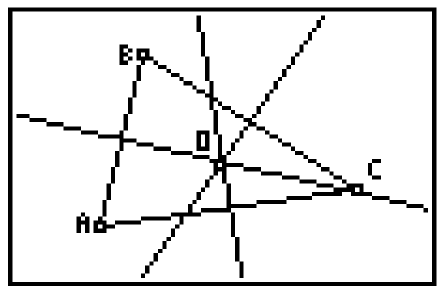
Construct a triangle and its circumcenter.

Clear the previous construction.

Draw $\triangle ABC$ in the center of the screen.

Construct the perpendicular bisector of each side of $\triangle ABC$.

Construct a point O where the perpendicular bisectors intersect each other.



Exploration

Use various tools (for example, measurement and circle tools) to investigate the properties of the circumcenter.

Drag the vertices and sides of the triangle and observe which properties of a circumcenter are true for any triangle and which are true only for certain types of triangles.

Questions and Conjectures

Describe at least two properties of a circumcenter that you observed.

Part III: Orthocenter of a Triangle

Construction

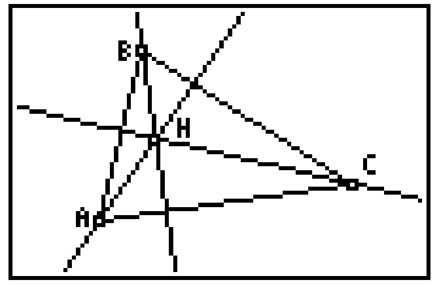
Construct a triangle and its orthocenter.

Clear the previous construction.

Draw ΔABC in the center of the screen.

Construct the three altitudes of ΔABC. (See Activity 18 for instructions on constructing an altitude.)

Construct a point H where the altitudes intersect each other.



Exploration

Use various tools (for example, measurement and circle tools) to investigate the properties of the orthocenter.

Drag the vertices and sides of the triangle and observe which properties of an orthocenter are true for any triangle and which are true only for certain types of triangles.

Questions and Conjectures

Describe at least two properties of an orthocenter that you observed.

Extension

Investigate the relationship among the different triangles that can be formed using the vertices of the original triangle and point H, the orthocenter of the original triangle. Make a conjecture about the relationship among these points (called an *orthic set*) and be prepared to demonstrate.

Part IV: Incenter of a Triangle

Construction

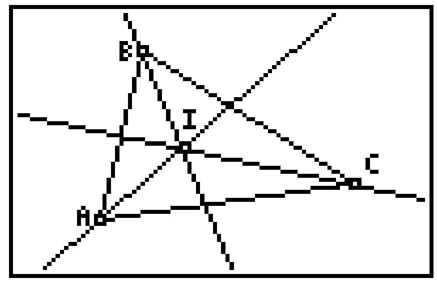
Construct a triangle and its incenter.

Clear the previous construction.

Draw ΔABC in the center of the screen.

Construct the angle bisector of each angle of ΔABC. (See Activity 18 for instructions on constructing an angle bisector.)

Construct a point I where the angle bisectors intersect each other.



Exploration

Use various tools (for example, measurement and circle tools) to investigate the properties of the incenter.

Drag the vertices and sides of the triangle and observe which properties of an incenter are true for any triangle and which are true only for certain types of triangles.

Questions and Conjectures

Describe at least two properties of an incenter that you observed.

Extension

The incenter can be used in a construction of a point known as the *Gergonne point*. Research the Gergonne point and write the steps of a construction that will find this point. Be prepared to demonstrate your construction.

Teacher Notes

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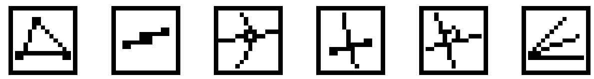
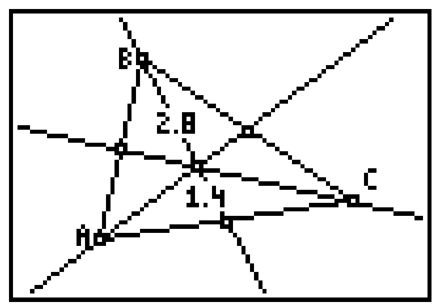


Part I: Centroid of a Triangle

Answers to Questions and Conjectures

Describe at least two properties of a centroid that you observed.



The centroid divides each median into two segments in the ratio of 2 to 1. The three medians divide the triangle into six smaller triangles that are of equal area, but not necessarily congruent.

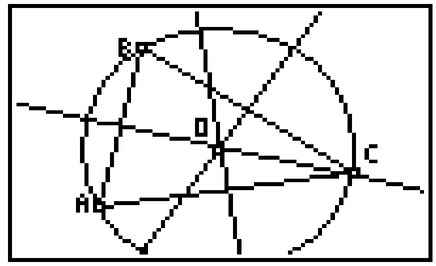
The centroid is the balancing point, or center of gravity, for the triangle. Students could verify this using a triangular piece of cardboard balanced on a pencil point. On the reverse side of the triangle, find the centroid using a compass and a straight edge. Compare the two points by pushing the point through the cardboard. Balance the cardboard triangle at the centroid and see what happens.

Part II: Circumcenter of a Triangle

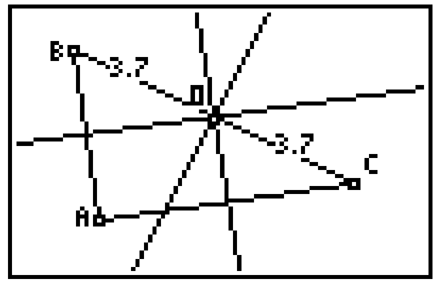
Answers to Questions and Conjectures

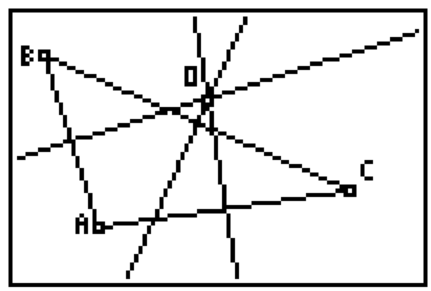
Describe at least two properties of a circumcenter that you observed.

The circumcenter is the center of the circle that circumscribes the triangle. Draw a circle using the circumcenter as the center and any vertex as a radius. This circle passes through all three vertices of the triangle. The distance from the circumcenter to any vertex is the same since these distances represent radii of the circle.

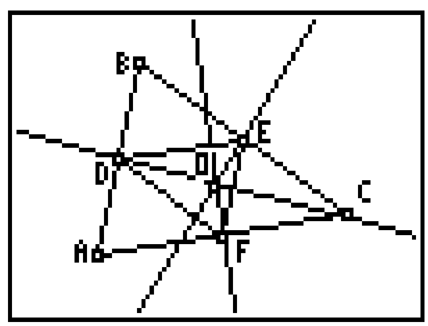


For an acute triangle, the circumcenter is in the interior of the triangle; for a right triangle, it is on the midpoint of the hypotenuse; and for an obtuse triangle, the circumcenter is a point exterior to the triangle.





Drawing the midsegments of ΔABC produces a new triangle, ΔDEF, such that perpendicular bisectors of ΔABC are the altitudes of ΔDEF. This means that the circumcenter of ΔABC is the orthocenter of ΔDEF.

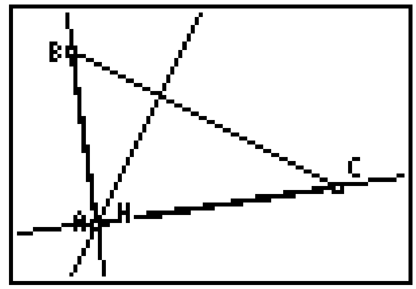


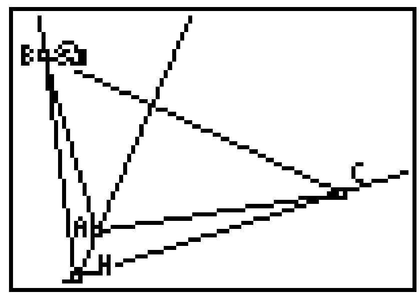
Part III: Orthocenter of a Triangle

Answers to Questions and Conjectures

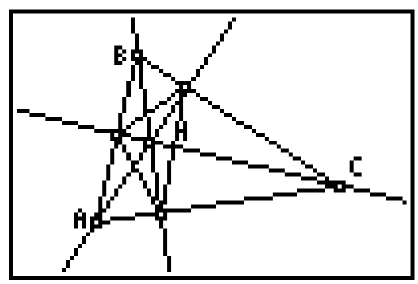
Describe at least two properties of an orthocenter that you observed.

The orthocenter (historically labeled *H*) is in the interior of an acute triangle, exterior to an obtuse triangle, and at the vertex of the right angle of a right triangle.





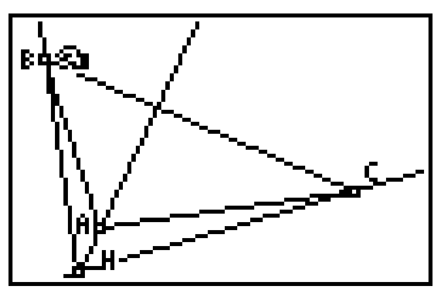
Point *H*, the orthocenter of $\triangle ABC$, is also the *incenter* of the triangle with vertices at the feet of the altitudes of a triangle. This triangle is called the *orthic triangle*.



Answers to Extension

Investigate the relationship among the different triangles that can be formed using the vertices of the original triangle and point *H*, the orthocenter of the original triangle. Make a conjecture about the relationship among these points (called an orthic set) and be prepared to demonstrate.

If the three vertices and orthocenter are taken together as a set of four points, a triangle formed using any combination of three of these points will have the fourth point as its orthocenter. In the figure, point *H* is the orthocenter of $\triangle ABC$ and point *C* is the orthocenter of $\triangle AHB$. This set of four points is called an *orthic set*.

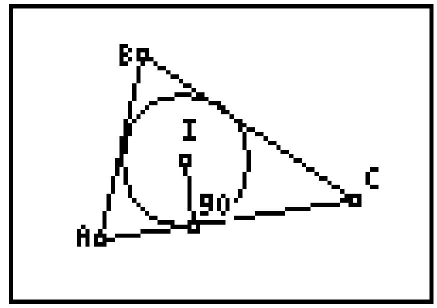


Part IV: Incenter of a Triangle

Answers to Questions and Conjectures

Describe at least two properties of an incenter that you observed.

The incenter is the point that is equidistant from the three sides of the triangle. The incenter is the center of a circle (called the *incircle*) that is internally tangent, or inscribed, inside the triangle. Since an angle bisector is the set of points equidistant from the sides of an angle, the incenter is the point that is equidistant from all three sides of the triangle. Construct a perpendicular line from the incenter to any side of the triangle. The intersection of this perpendicular line and the triangle defines the radius of the incircle and one point of tangency. You can use this perpendicular distance to the incenter to draw the incircle.



Extension

The incenter can be used in a construction of a point known as the *Gergonne point*. Research the Gergonne point and write the steps of a construction that will find this point. Be prepared to demonstrate your construction.

Construct the incircle as described above. Construct segments through the incircle's three points of tangency with the opposite vertices of the triangle. The point where these three segments intersect is the Gergonne point.