

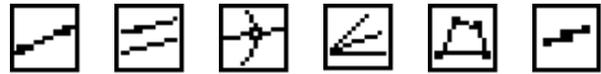
## Activity 24

### Angle Bisectors and Medians of Quadrilaterals

#### Objectives

- To investigate the properties of quadrilaterals formed by angle bisectors of a given quadrilateral
- To investigate the properties of quadrilaterals formed by medians of a given quadrilateral

#### Cabri® Jr. Tools



#### Introduction

This investigation is an extension of Activity 18, Altitude, Median, and Angle Bisector of a Triangle. You will be constructing some of the same lines in quadrilaterals that you did in triangles to explore the properties that appear to be true for four-sided figures. You may find it necessary to review the definitions and properties of special quadrilaterals in order to construct the figures and explain the outcomes.

This activity makes use of the following definitions:

**Parallelogram** — a quadrilateral with two pairs of parallel sides

**Angle bisector** — a line that passes through the vertex of an angle, forming two congruent angles

**Median** — a segment drawn between a vertex and the midpoint of a non-adjacent side

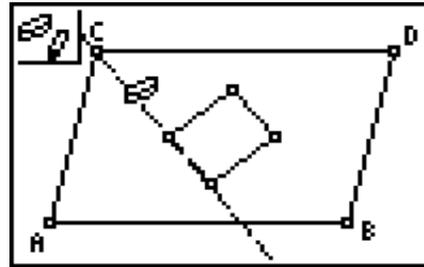
#### Part I: Angle Bisectors of Parallelograms

##### Construction

Construct a parallelogram and its four angle bisectors.

-   Draw a horizontal segment  $\overline{AB}$ . Construct a second segment  $\overline{AC}$ .
-  Construct a line parallel to  $\overline{AB}$  that passes through and is defined by point  $C$ . Construct a line parallel to  $\overline{AC}$  that passes through and is defined by point  $B$ .
-   Construct point  $D$  at the intersection of the parallel lines that you constructed.

-  Verify that you have constructed a parallelogram by dragging a side or vertex to see that the figure remains a parallelogram.
-  Construct the angle bisectors of each of the four interior angles of the parallelogram.
-  Construct the quadrilateral formed by the intersections of the angle bisectors.
-  Hide the angle bisectors and parallel lines used to construct the quadrilaterals to better view the new quadrilateral.



### Exploration

-   Drag a vertex and/or side of the original parallelogram and observe the new quadrilateral formed by the angle bisectors. Be sure to investigate what happens when the original parallelogram is a special parallelogram (rectangle, rhombus, or a square). Use various measurement tools to investigate the relationships you believe exist.

### Questions and Conjectures

1. Make a conjecture about the quadrilateral formed by the angle bisectors of a general parallelogram. Explain your reasoning and be prepared to demonstrate.
2. Make conjectures about the quadrilateral formed by the angle bisectors of special parallelograms (rectangle, rhombus, and square). Explain your reasoning and be prepared to demonstrate.

### Extension

Make conjectures related to the quadrilaterals formed by the angle bisectors for the following quadrilaterals:

- A trapezoid
- An isosceles trapezoid
- A kite
- A general quadrilateral

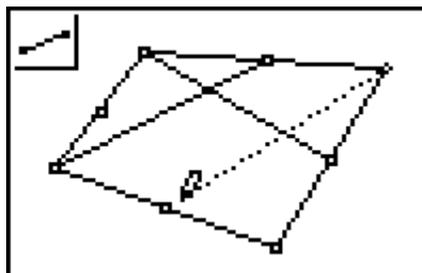
## Part II: Medians of Quadrilaterals

A median of a quadrilateral is a segment drawn between a vertex and the midpoint of a non-adjacent side. Unlike triangles, quadrilaterals have two non-adjacent sides for each vertex. The medians are always constructed using a midpoint only once. This is most easily accomplished by constructing the median to the next non-adjacent side as you move in one direction around the quadrilateral (that is clockwise or counter-clockwise).

### Construction

#### I. Construct the medians of a general quadrilateral.

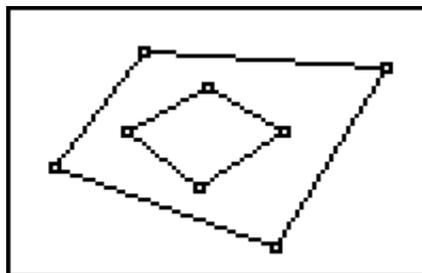
-  Construct a general quadrilateral.
-  Construct the midpoint of each side of the quadrilateral.
-  Construct segments connecting each vertex to the midpoint of the side opposite it. The segments must be drawn in either a clockwise, or counter-clockwise manner.



*Note: Only three clockwise medians are shown.*

#### II. Construct the quadrilateral formed by the medians.

-  Construct the quadrilateral formed by the intersection of the medians.
-  Hide the medians and the midpoints to better view the new quadrilateral.



### Exploration

-   Drag the vertices and/or sides of the original quadrilateral to form special quadrilaterals (for example kites, trapezoids, parallelograms, and so forth) and note what happens to the quadrilateral formed by the medians. Use various measurement tools to investigate the relationships you believe exist.
-   Calculate the ratio of the area of the original quadrilateral to the area of the quadrilateral formed by the medians. Investigate this ratio as you change the original quadrilateral as described above.

### ***Questions and Conjectures***

1. For which special quadrilaterals does the quadrilateral formed by the medians create a special quadrilateral? Explain why this is true.
2. Make a conjecture about the relationship between the areas of the two quadrilaterals.
3. Do your answers to the previous questions depend on whether you constructed the medians in a clockwise or counter-clockwise direction? Explain how you know.

### ***Extension***

Consider the medians of a parallelogram. Justify the area relationship between a parallelogram and the quadrilateral formed by the medians.

## Teacher Notes



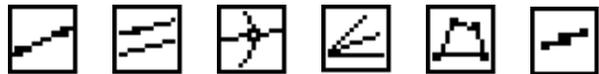
### Activity 24

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### Additional Information

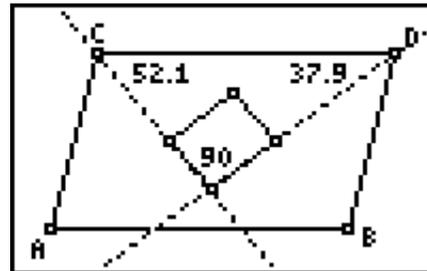
It is recommended that you complete Activity 21, Constructing Quadrilaterals, prior to completing this activity.

### Part I: Angle Bisectors of Parallelograms

#### Answers to Questions and Conjectures

1. Make a conjecture about the quadrilateral formed by the angle bisectors of a general parallelogram. Explain your reasoning and be prepared to demonstrate.

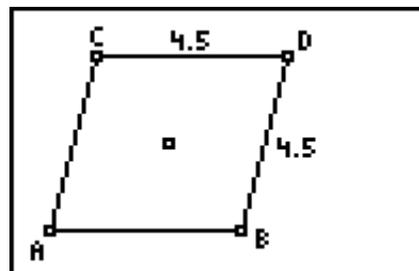
For the general parallelogram, the resulting quadrilateral is a rectangle. Since the consecutive angles of a parallelogram are supplementary and the measures of opposite angles are equal, each pair of angle bisectors forms a triangle having two base angles that sum to  $90^\circ$ . Therefore, the angle bisectors intersect at right angles, forming a rectangle.



2. Make conjectures about the quadrilateral formed by the angle bisectors of special parallelograms (rectangle, rhombus, square). Explain your reasoning and be prepared to demonstrate.

For a rectangle, the resulting quadrilateral is a square. Because the angles of the rectangle are  $90^\circ$ , the triangles formed by the intersections of the angle bisectors are isosceles triangles. From this it can be shown that the sides of the rectangle are congruent and therefore, the rectangle is a square.

If the parallelogram is a square or rhombus, the angle bisectors of one angle will bisect the opposite angle. Therefore, the two unique angle bisectors will intersect at a single point, and no quadrilateral will be formed.

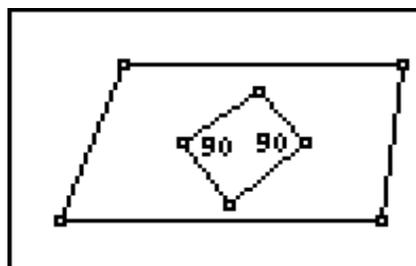


**Answer to the Extension**

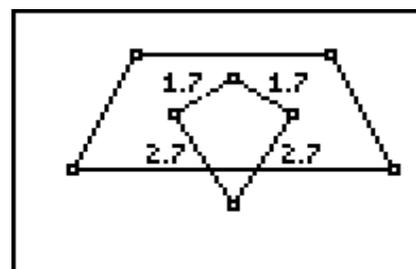
Make conjectures related to the quadrilaterals formed by the angle bisectors for the following quadrilaterals:

- A trapezoid
- An isosceles trapezoid
- A kite
- A general quadrilateral

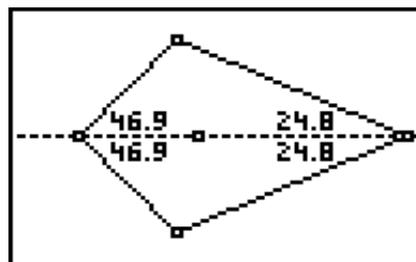
**Trapezoid:** The new quadrilateral formed has one pair of opposite right angles because a trapezoid has only one pair of parallel sides.



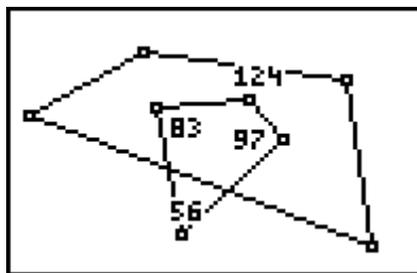
**Isosceles trapezoid:** The new quadrilateral is a kite.



**Kite:** No quadrilateral is formed since one of the angle bisectors bisects the opposite angle.



**General quadrilateral:** The quadrilateral formed is cyclic. A cyclic quadrilateral is a quadrilateral that can be inscribed in a circle. A special property of a cyclic quadrilateral is that opposite angles are supplementary. This is not true of all quadrilaterals. See Activity 26, Constructing Circles, to explore the construction of a circle that passes through four points.



## Part II: Medians of Quadrilaterals

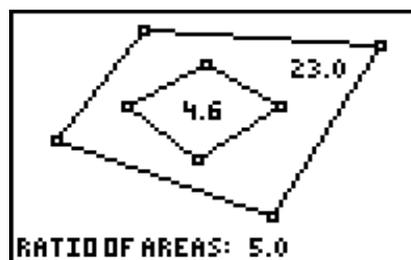
### Answers to Questions and Conjectures

1. For which special quadrilaterals does the quadrilateral formed by the medians create a special quadrilateral? Explain your reasoning.

If the original quadrilateral is a parallelogram, the quadrilateral formed by the medians is a parallelogram. This is true since opposite medians of a parallelogram will be parallel. If the original quadrilateral is a square, then the inner parallelogram is a square. This is true since the intersecting medians are perpendicular and all have equal lengths thus dividing the original square into four congruent trapezoids, four congruent triangles, and a square.

2. Make a conjecture on the relationship between the areas of the two quadrilaterals.

The ratio of the areas of the original quadrilaterals to the quadrilateral formed by the medians is 5:1. See the Extension for a visual representation.



3. Do your answers to the previous questions depend on whether you constructed the medians in a clockwise or counter-clockwise direction? Explain your reasoning.

The results do not change if you construct the clockwise medians or the counter-clockwise medians.

**Answers to Extension**

Consider the medians of a parallelogram. Justify the area relationship between a parallelogram and the quadrilateral formed by the medians.

The medians of a parallelogram divide the original parallelogram into one parallelogram, four congruent quadrilaterals, and four congruent triangles. The area relationship can be supported by rotating the triangles  $180^\circ$  around the midpoint of the corresponding side of the original parallelogram. The rotations join the triangles with the quadrilaterals forming parallelograms that are congruent to the parallelograms formed by the medians. These five congruent parallelograms have the same area as the original parallelogram. Therefore, the ratio of the area of the original parallelogram to the area of the parallelogram formed by the medians is 5:1.

