

Linear Programming

by – Christine Czapleski

Activity overview

This activity adds a twist to a traditional linear programming problem by using the features of the TI-Nspire calculator.

Concepts

Converting verbal problems into equations and inequalities.
Graphing inequalities
Finding points of intersection of linear equations
Finding the overlapping regions for linear inequalities
Distinguishing between the graphs of functions and relations

Teacher preparation

This would probably not be the first linear programming problem attempted by the students. It would be a great summary exercise.

Classroom management tips

Students should work with a partner or a group so that they can discuss the problem as they go along.

TI-Nspire Applications

Graphing Inequalities
Finding Intersection points
Moving the graph of an equation
Using a spread sheet

Step-by-step directions

Teacher Pages

The manufacture of sleeping bags makes a standard model and a sub-zero model. Each standard sleeping bag requires 1 labor-hour from the cutting department and 3 labor-hours from the assembly department. Each sub-zero sleeping bag requires 2 labor hours from the cutting department and 4 labor-hours from the assembly department. The maximum labor-hours available per week in the cutting department and the assembly department are 32 and 84, respectively. In addition, the distributor, because of demand, will not take more than 12 sub-zero sleeping bags per week. If the company makes a profit of \$50 on each standard sleeping bag and \$80 on each sub-zero sleeping bag, how many sleeping bags of each type should be manufactured each week to maximize the total weekly profit?

Organize the manufacturing requirements, objectives and restrictions in the table below.

| | Labor-hours per sleeping bag Standard Model | Labor-hours per sleeping bag Sub-Zero Model | Maximum Labor-Hours Available per week |
|-------------------------|--|--|--|
| Cutting Department | 1 | 2 | 32 |
| Assembly Department | 3 | 4 | 84 |
| Profit per sleeping bag | \$50 | \$80 | |

The objective of management is to decide how many of each model of sleeping bag should be produced to maximize the profit.

Choose variable to represent each of the **decision** quantities.

$x =$ _____ the number of standard models _____

$y =$ _____ the number of sub-zero models _____

Write an equation to represent the total profit assuming all sleeping bags manufactured are sold.

Profit: $P = 50x + 80y$ _____ (This is the **Objective function**)

Mathematically, the management needs to decide the values for the **decision variables** (x, y) that achieve its objective, that is maximizing the **objective function**.

Any manufacturing company has manufacturing limits imposed by available resources, plant capacity, demand, and so forth. These limits are referred to as **constraints**.

Use the contents of the above table to write two inequalities which represents the manufacturing constraints.

$X+2y \leq 32$ Cutting department

$3x + 4y \leq 84$ Assembly department

Use the information in the problem to write an inequality which represents the demand constraint.

$y \leq 12$

It is not possible to manufacture a negative number of sleeping bags; thus there are two inequalities which represent the **nonnegative constraints**.

$$x \geq 0$$

$$y \geq 0$$

Use your TI *nspire* to graph the constraints. The overlapping region is called the **feasible region**.

Begin with a new TI-Nspire document, and create a Graphs and Geometry page.

Move the cursor to the entry line and press ENTER to activate it. Clear to remove the = sign and replace it with <= followed by your expression.

The $f1(x)$ changes to y when an inequality is graphed, what is the significance of this change?

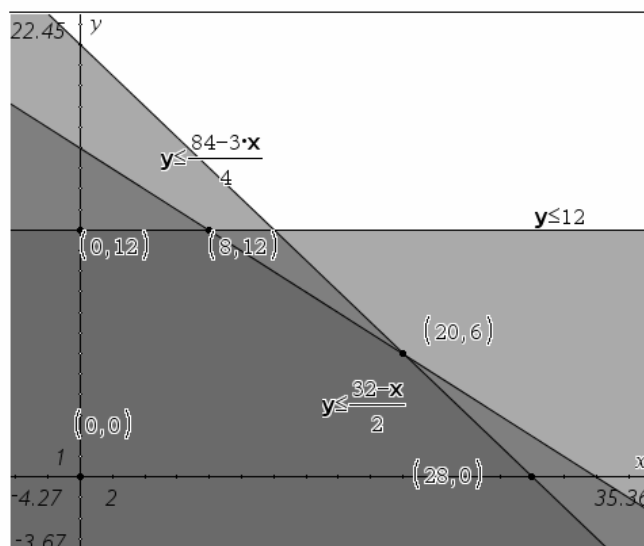
_____ *The region is a relation and not a function; therefore functional notation cannot be used.* _____

Continue to enter the next two constraints and then adjust the window to show the first quadrant to accommodate the nonnegative constraints. The window is found MENU-4:WINDOW-1:WINDOW SETTINGS.

Find the corner points of the feasibility region using

Two of the intersection points can be found using: MENU-6:POINTS&LINES-3:INTERSECTION POINTS.

The other three points can be found using: MENU-6:POINTS&LINES-2: POINT ON.



To estimate which point will provide the maximum profit do the following:

Move the cursor to the entry line and enter the object function without the y-intercept.
 Move the cursor back to the graph screen and move the line vertically until it intersects the feasibility region in just one point.

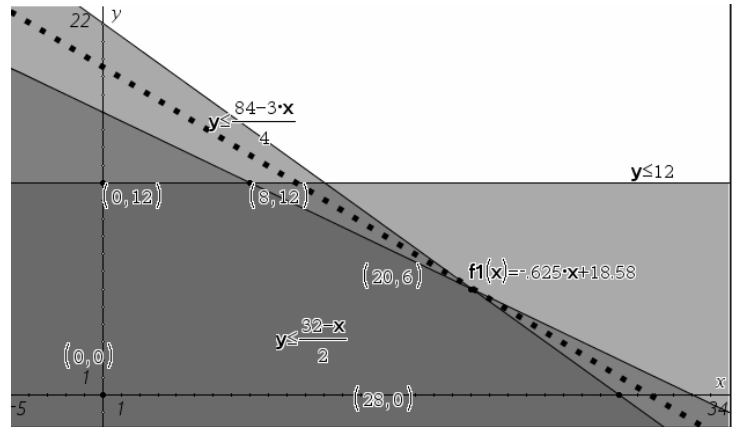
$$P = 50x + 80y$$

$$P - 50x = 80y$$

$$\frac{P}{80} - \frac{50}{80}x = y$$

Graph: $f_1(x) = -0.625x$

(Select MENU-1.Tools-3.Attributes. Move the cursor to the line and press ENTER. This will allow you to change the appearance of the line.)



It is necessary to use the decimal form of the fraction in order to be able to move the line.

What is your estimate of the point that will maximize the profit? (20,6)

Calculate the maximum profit at this point. \$1480

To check your estimate, add a new page: MENU-3:Lists and Spreadsheets.

Enter the coordinates of the five corner points into the A and B columns of the spreadsheet.

In the white box next to the A, enter the word "standard". In the white box next to the B, enter the word "subzero".

In column C, go to the gray formula box and enter the object function in the following manner.
 $= 50 *VAR-3LINK TO-standard + 80*VAR-3LINK TO-subzero.$

Now look at the generated values and verify that the ordered pair that produces the maximum profit is the same ordered pair as you observed on the graph.

| | A standard | B subzero | C | D |
|----|------------|-----------|------------------|---|
| ◆ | | | =50*'standard+80 | |
| 1 | 0 | 0 | 0 | |
| 2 | 0 | 12 | 960 | |
| 3 | 8 | 12 | 1360 | |
| 4 | 20 | 6 | 1480 | |
| 5 | 28 | 0 | 1400 | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |

Answer the following.

How many standard sleeping bags should the company produce to maximize the profit?

They should make 20 standard sleeping bags.

How many sub-zero sleeping bags should the company produce to maximize the profit?

They should make 6 subzero sleeping bags.

What is the maximum profit for the company?

The maximum profit for the company is \$1480.

Assessment and evaluation

Answers are provided in the teacher document. Teachers can determine success of the activity if students successfully complete the exercise

Activity extensions

To determine if the students understand both the mathematics and the calculator procedure repeat the activity with any linear programming problem from an Algebra II text.

Student TI-Nspire Document

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| Assembly Department | | | |
| | | | |
| Profit per sleeping bag | | | |

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Choose variable to represent each of the decision quantities.

$x =$ _____

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Profit: $P =$ _____ (This is the Objective function)

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Move the cursor back to the graph screen, grab and move the line vertically until it intersects the feasibility region in just one point.

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Answer the following.

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How many sub-zero sleeping bags should the company produce to maximize the profit?

What is the maximum profit for the company?