Vertical and Phase Shifts

ID: 8316

Time required
45 minutes

Activity Overview

In this activity, students explore vertical and phase shifts of sine and cosine functions, after a brief review of period and amplitude. Students will manipulate sliders to change the values of parameters in trigonometric functions; they will determine the effect that each change has upon the shape of the graph. They will then use this knowledge to write a sine function as a cosine function.

Topic: Trigonometric Functions

- *Analyze and predict the effects of changes in A, B and C on the graphs of f(x) = Asin(Bx+C), f(x) = Acos(Bx+C), and f(x) = Atan(Bx+C) and interpret A, B, and C in terms of amplitude, period, and phase shifts.*
- *Approximate the amplitude, frequency, and phase shift of the primary trigonometric functions by graphing.*

Teacher Preparation and Notes

- *Students should already be familiar with the graphs of the sine and cosine functions, and they should also have some experience in determining the period (in radians) and identifying the amplitude of a trigonometric function.*
- *The graphs of trigonometric functions that students will view on their handhelds have been constructed such that the horizontal scale is in multiples of $\pi/2$. This scale is fixed—it cannot be adjusted by dragging the tick marks. The windows may be adjusted by a few units in any direction, but make sure students do not stray too far from the origin.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire™ document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8316" in the keyword search box.***

Associated Materials

- *VerticalandPhaseShifts_Student.doc*
- *VerticalandPhaseShifts.tns*

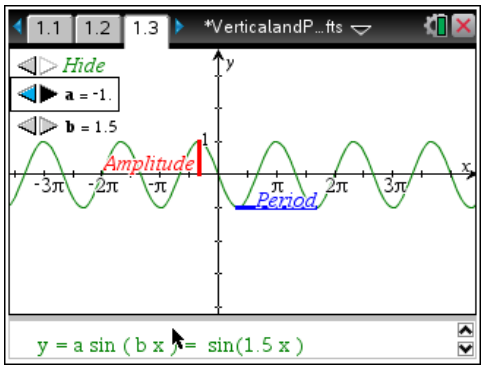
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Can You Make My Graph? (TI-Nspire™ technology) — 9161*
- *Find That Sine (TI-Nspire™ technology) — 9734*

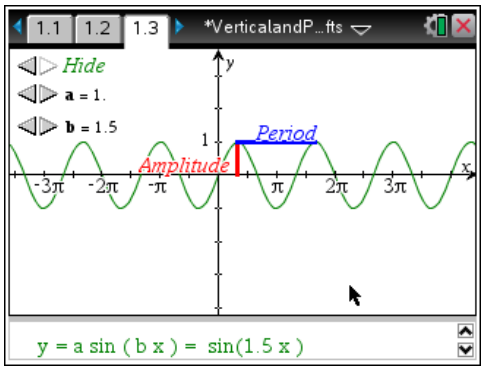
Problem 1 – Amplitude and Period

This problem allows students to quickly review the amplitude and period of a function of the form $f(x) = a \sin(bx)$. Clicking the slider for a, students should conclude that the sine curve is vertically stretched by a factor of $|a|$. Thus, amplitude = $|a|$. Be sure to ask what effect the sign of a has on the graph (if a is negative, then the curve is reflected over the x-axis).



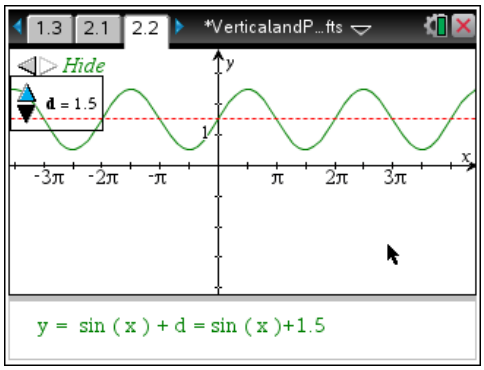
Clicking the slider for b, students will find that the value of b affects the horizontal stretch of this function and thus changes the period of the function.

If students do not immediately recall how the value of b is related to the period of the function, have them set the value of b to 0.25, 0.5, 1, and 2 and identify the period for each (8π, 4π, 2π, and π, respectively). After some examination, students should be able to identify the relationship: period = $\frac{2\pi}{b}$.



Problem 2 – Vertical Shift

The parameter d in the graph of $f(x) = \sin(x) + d$ is considered before c because it is relatively straightforward. Before students click on the slider, ask them to predict what will happen by first considering a different function, $y = x^2$. Ask them how to obtain the graph of $y = x^2 + 3$ from the graph of $y = x^2$ (translate the graph up three units). Students should confirm that the vertical shift is equal to this parameter; that is, vertical shift = d.

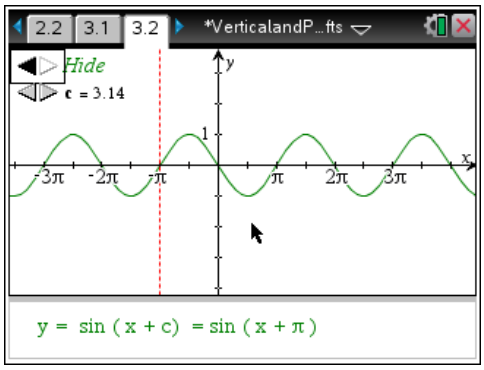


TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

See Note 1 at the end of this lesson.

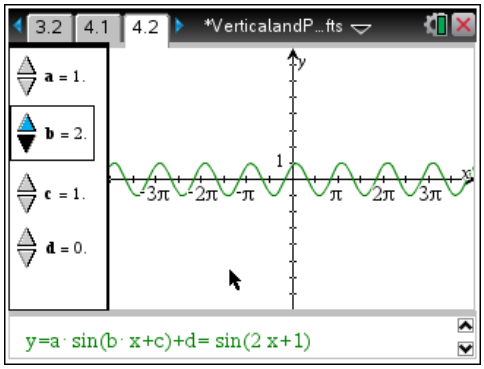
Problem 3 – A Simple Phase Shift

The graph of $f(x) = \sin(x + c)$ is shown on page 3.2. Students might predict that a change in c will result in a horizontal shift a certain number of units (here called a phase shift), but how that number of units relates to c will not be immediately clear. This is explored in Problem 4.



Problem 4 – Combining Transformations

While students may be wondering why they cannot declare "phase shift = –c," have them again consider the graph of $y = x^2$. Ask them how to obtain from it the graph of $y = (x + 3)^2$ (translate three units to the left). Then display the equations $y = [4(x + 3)]^2$ and $y = (4x + 3)^2$, and elicit from students that only the former represents a shift three units to the left. At this point students should realize that the phase shift of $f(x) = a \sin(bx + c) + d$ depends on <u>two</u> parameters: b and c.



To establish exactly how b and c determine the phase shift, students may click the sliders on page 4.2. Encourage students to make a conjecture about the relationship on their own, but if they need help, have them consider the phase shift when $b = 1$ and $c = 2$, when $b = 2$ and $c = 1$, and when $b = 2.5$ and $c = –5$. (It is –2, –0.5, and 2, respectively.) Examining these values, students should conclude that phase shift = $-\frac{c}{b}$.

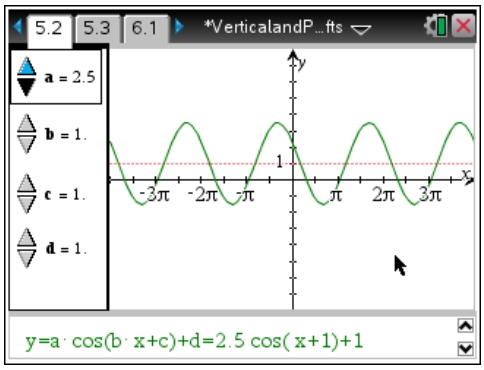
<u>Note</u>: It is easiest to identify this relationship if parameters a and d are left as initially set ($a = 1$ and $d = 0$). After the relationship is determined, clicking the sliders can verify that neither a nor d affects the phase shift.

On page 4.3 students are asked to summarize their results from Problems 1–4. Make sure students have correctly completed this page before proceeding further.

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

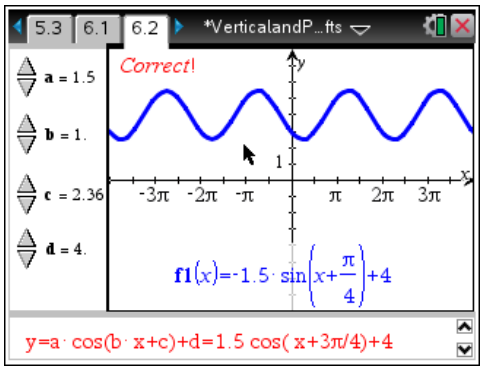
Problem 5 – Cosine Function

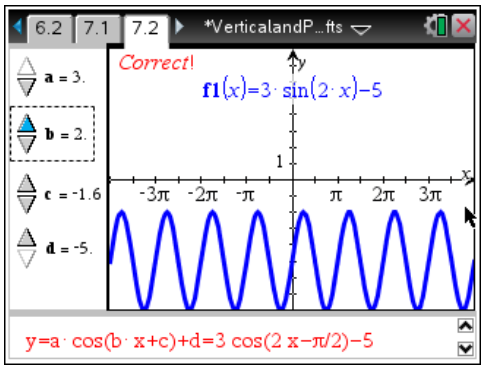
On page 5.1, students are asked to verify that all results from Problems 1-4 hold for the cosine function and summarize their findings.



Problems 6 and 7– Putting It All Together

Finally, students will apply what they have learned about vertical and phase shifts. They are given the equations and graphs of two sine functions and asked to find equations of cosine functions that coincide. Students should observe that the values of *a*, *b*, and *d* are the same for each sine/cosine matching pair; the only difference occurs in the value of *c*. Because these functions are periodic, there are infinitely many equations that satisfy each condition. Be sure to check students' equations.





TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 3 at the end of this lesson.

TI-Nspire™ Navigator™ Opportunities

Note 1
Problems 1 and 2, *Quick Poll*
You may want to send a Quick Poll to assess students' understanding of how changes in various parameters affect the graphs.

Note 2
Problem 4, *Quick Poll*
You may want to send a Quick Poll to determine whether or not students understand the effect of applying more than one transformation at a time.

Note 3
Question 10, *Class Capture*
Take a Class Capture once all students have determined a cosine function to match the given sine function. Scroll through the students' screens, asking what is common about the functions. Students should see that there is more than one correct answer. Discuss why there are multiple correct answers.