

What's the Spin? - Discover Properties of Geometric Rotations

Geometry Major Topics: Rotations and their relation to reflections

NCTM Principles and Standards:
Content Standards

Geometry

Apply transformations and use symmetry to analyze mathematical situations.

Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices.

Use various representations to help understand the effects of simple transformations and their compositions.

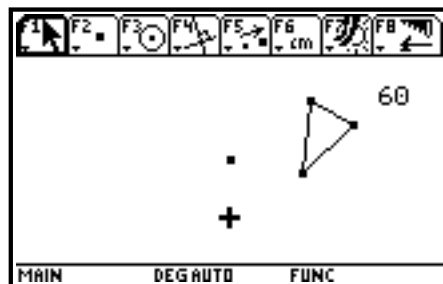
Objectives: This lesson has students discover properties of rotations. Using the CABRI Geometry II Application for the TI-89 calculator, students rotate a triangle around a point and through a specified angle. The students further discover that a rotation is the composition of two reflections over intersecting lines. The relationship between these intersecting lines, the center of rotation, and the angle of rotation is also discovered. This lesson can be used as a teacher directed activity or can be given to the students as a discovery activity. If the students are familiar with the CABRI Geometry II application, this activity should take one 50- minute class period.

Materials: TI-89 Graphing Calculator, CABRI Geometry II Application

Prerequisites: Students should have a working knowledge of the CABRI Geometry II application and should have previously studied reflections.

Procedure:

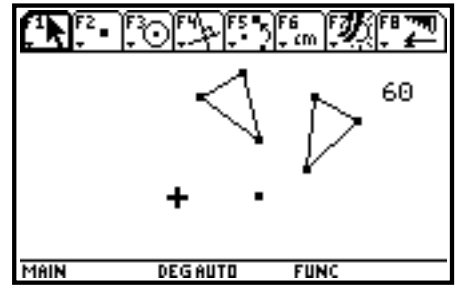
Activity 1: Open the CABRI Geometry II application on the TI-89 by pressing the APPS button and choosing Cabri Geometry. Open a NEW file and choose an appropriate variable name, such as rotation. Press F3 and choose **3:Triangle**. Construct an acute scalene triangle on the right side of the screen. Press F2 and choose **1:Point**. Construct a point near the middle of the screen. Press F7 and choose **6:Numerical Edit**. Move the cursor to a location in the upper right corner of the screen and press ENTER. This creates a numerical edit box. Type in the number 60.



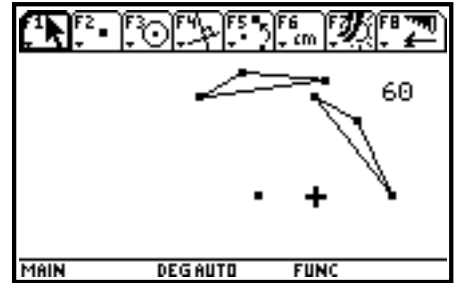
Press F5 and choose **2:Rotation**. Move the cursor to the triangle; when the message "rotate this triangle" appears, press ENTER. This establishes the triangle as the object to be rotated. Move the cursor to the point in the middle of the screen; when the message "around this point" appears, press ENTER. This establishes the point as the

center of rotation. Move the cursor to the number; when the message "using this angle" appears, press ENTER.

The original triangle has been rotated around the center by an angle of 60 degrees. It is possible that the entire image triangle does not appear on the screen. Try dragging the center of rotation point until both triangles appear. Observe that the rotation has been in a counterclockwise direction.



Press F1 and choose **1:Pointer**. Drag one of the vertices of the original triangle. Observe what happens to the image triangle.

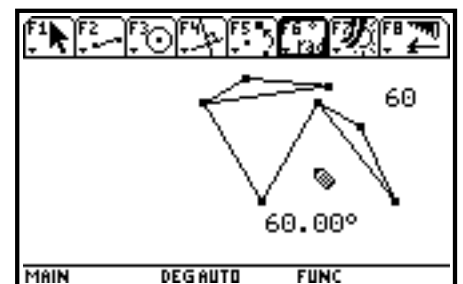


It has been determined that the image triangle can be altered by altering the original triangle and that the position of the image triangle can be changed by dragging the center of rotation point to various locations. Now experiment with trying to change the position of the image triangle by changing the rotational angle. Press F7 and choose **6:Numerical Edit**. Move the cursor to the number; when the message "this number" appears, press ENTER. This will create an edit box around the number, allowing you to change the number. Delete the number 60 and type in the number 100. Observe the new position of the image triangle. Experiment with different numbers including 180, numbers larger than 180, 360, numbers larger than 360, and negative numbers. Make conjectures about the results of rotating the triangle through these various angles.

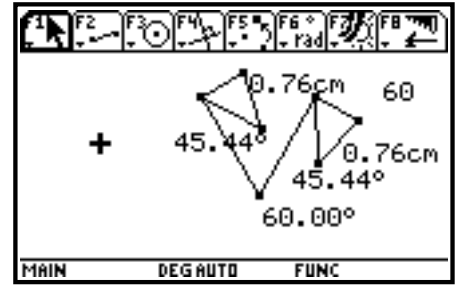
Teacher Note: The conjecture should be that a rotation by a positive angle rotates in a counterclockwise direction. A rotation by a negative angle rotates in a clockwise direction. A rotation by 180 degrees accomplishes a half-turn around the point. A rotation by 0 degrees or 360 degrees produces equivalent results, a rotation back onto the original triangle. Numbers larger than 360 revert back to their equivalents modulo 360.

Switch the number back to 60. The triangle has been rotated 60 degrees counterclockwise around the center of rotation point. Determine the location of the 60 degree angle.

Teacher Note: The angle can be located by constructing a segment from the center of rotation point to a vertex of the original triangle and a segment from the center of rotation point to the corresponding vertex of the image triangle. This forms an angle whose measure corresponds to the number in the numerical edit box.

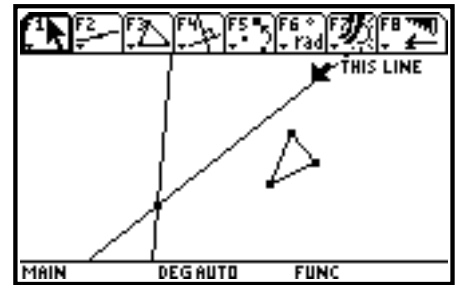


Press F6 and choose **1:Distance & Length**. Measure the distance between two vertices of the original triangle and then measure the distance between the corresponding vertices of the image triangle. Press F6 and choose **3:Angle**. Measure one of the angles of the original triangle and then measure the corresponding angle of the image triangle. Press F1 and choose **1:Pointer**. Drag each of the vertices of the original triangle. Make a conjecture about the lengths of rotated segments and the measures of rotated angles.

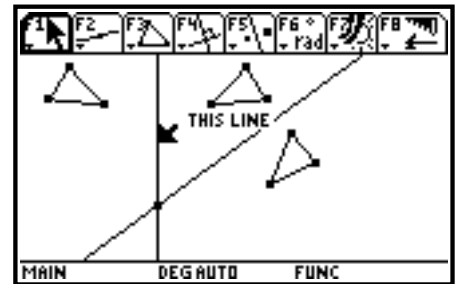


Teacher Note: The conjecture should be "rotations preserve distances and angle measures."

Activity 2: Press F8 and choose **8:Clear All**. Press ENTER to clear the screen for the next construction. Press F3 and choose **3:Triangle**. Construct an acute scalene triangle on the right side of the screen. Press F2 and choose **4:Line**. Construct two intersecting lines that do not intersect the triangle.



Press F5 and choose **4:Reflection**. Move the cursor to the triangle; when the message "reflect this triangle" appears, press ENTER. This establishes the triangle as the object to be reflected. Move the cursor to the line closest to the triangle; when the message "with respect to this line" appears, press ENTER. This will reflect the triangle over this line. Drag the original triangle or the intersecting lines so that the image triangle is between the two lines. In a similar manner, reflect the image triangle over the second of the intersecting lines. Your figure should be similar to the screen at the right.



Observe the original triangle and the second of the image triangles. Could this image have been accomplished by one transformation instead of reflecting twice over intersecting lines? Make a conjecture as to how this could be accomplished.

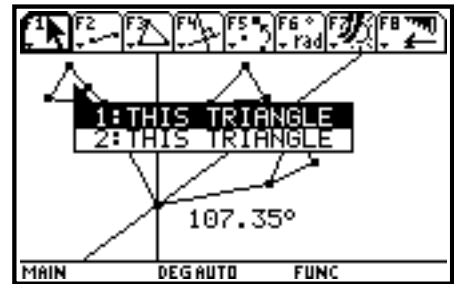
Teacher Note: The same effect could have been achieved with a rotation.

Confirm this conjecture by finding the center of rotation and the angle of rotation that accomplishes this rotation.

Teacher Note: The center of rotation is the intersection point of the two lines. The angle of rotation can be established by constructing a segment that connects the point of intersection of the two lines to a vertex of the original triangle and a segment that connects the point of intersection to the corresponding vertex of the second image triangle. Measure this angle.

Confirm this is the correct rotation by rotating the original triangle around the point of intersection of the two lines using the measured angle.

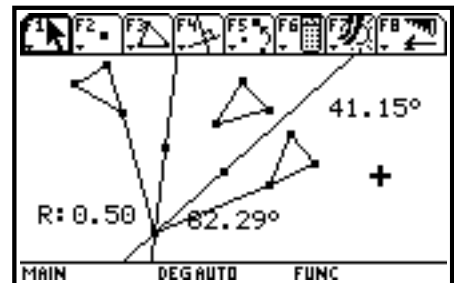
Teacher Note: Although the screen appears not to have changed, a rotation has occurred. To confirm the image of the rotation is the same as the second image of the composition of the two reflections, press F1 and choose **1:Pointer**. Move the cursor to the second image triangle and press ENTER. A message will appear asking if you wish to choose the triangle created by the two reflections or the triangle created by the rotation.



Activity 3: The figure from Activity 2 will be used for this activity. Drag any of the vertices of the original triangle to various locations on the screen. Next drag the entire triangle. Finally, drag either of the reflecting lines. While performing these activities, observe the angle between the reflecting lines and the rotational angle. Make a conjecture as to their relationship.

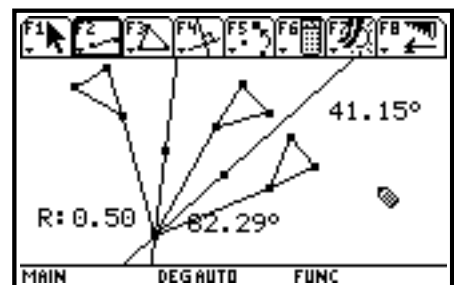
Teacher Note: The conjecture should be "the two reflecting lines form an angle that is one-half the measure of the rotation angle".

Confirm this conjecture by pressing F6 and choosing **3:Angle**. Measure the angle between the intersecting lines. The displayed measure of the angle between the intersecting lines may not be exactly one-half the measure of the rotation angle. This discrepancy is due to the precision of our measurement. To confirm the conjecture, press F6 and choose **6:Calculate**. Calculating the ratio of the two angle measurements will confirm the conjecture.



Based on your knowledge of reflections, explain why the conjecture must be true.

Teacher Note: Constructing a third segment from the intersection point of the lines to the corresponding vertex of the first image triangle will explain the conjecture. Since reflections preserve angle measures, two pairs of congruent angles are formed. The angle between the lines is the union of two angles, one angle from each pair; the rotational angle is the union of all four angles.



Extension: Determine if the composition of two reflections is commutative. In other words, will the reflection image remain the same if the triangle is first reflected over the farther line and then that image triangle is reflected over the closer line?

Teacher Note: The composition of two reflections over intersecting lines is not a commutative geometric operation. This can be observed by doing the reflections in the opposite order. The images are not the same. The "new" angle of rotation will still have twice the measure of the angle between the two intersecting lines; but one rotation will be clockwise, while the other is counterclockwise. The composition will be commutative in only one case. This occurs if the two intersecting lines are perpendicular. The angle between the lines is 90 degrees and the angle of rotation is 180 degrees. The image will remain the same whether the rotation is in a clockwise or counterclockwise direction.

Ray Klein
Glenbard West High School
Glen Ellyn, IL
RKlein9019@aol.com

Ilene Hamilton
Adlai Stevenson High School
Lincolnshire, IL
ihamilto@district125.k12.il.us