EXPLORATIONS™

Activity 26

Objectives

- To apply construction techniques to generate circles that pass through a given number of points
- To investigate the number of points needed to determine a unique circle in a plane

Cabri® Jr. Tools

Constructing Circles

Introduction

Given any two points, can you construct a circle passing through them? If you can, is it the only circle that can be constructed? Can you construct a circle passing through three points or four points? In this activity, you will investigate the construction of circles that pass through a given number of points. You will also investigate the number of points needed to generate a unique circle.

Construction

Costruct the following (each as a separate Cabri Jr. figure) using the given information.

Draw two points and construct a circle that passes through those points. If possible construct more than one circle that passes through both of the given points.

Draw three points and construct a circle that passes through those points. If possible construct more than one circle that passes through all of the given points.

Draw four points and construct a circle that passes through those points. If possible construct more than one circle that passes through all of the given points.

Exploration

Drag the given points and the defining points of the circle(s) to ensure that the circle(s) always passes through the given points.

Questions and Conjectures

1. Write the steps you used to construct each of the circles. State any necessary conditions for each construction. Be prepared to demonstrate that your construction works.

2. How many points are needed to determine a unique point that can be used as a center of a circle that passes through all of the given points? Explain your reasoning and be prepared to demonstrate.

Teacher Notes

Activity 26

Constructing Circles

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- To apply construction techniques to generate circles that pass through a given number of points
- To investigate the number of points needed to determine a unique circle in a plane

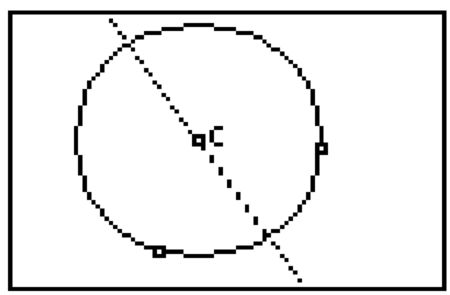
Cabri® Jr. Tools

Additional Information

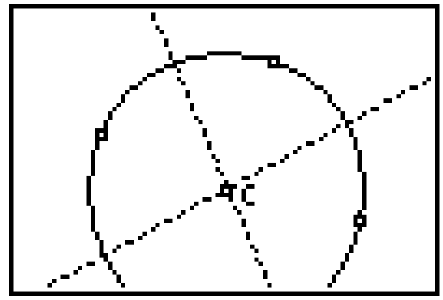
Unlike the other activities in this book, the Cabri Jr. tool list does not include a list of the tools that the students are likely to use in this activity.

Answers to Questions and Conjectures

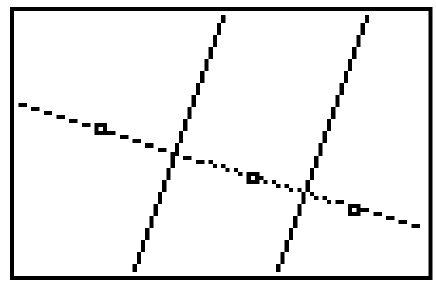
1. Write the steps you used to construct each of the circles. State any necessary conditions for each construction. Be prepared to demonstrate that your construction works.



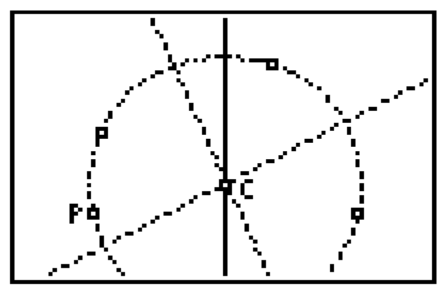
Passing through two given points — One technique for constructing a circle that passes through two fixed points simultaneously is to construct the perpendicular bisector between the two points. Place the center *C* of the circle anywhere on this perpendicular bisector and complete the circle by attaching it to either of the two fixed points. The circle will also pass through the other fixed point because of the Perpendicular Bisector Theorem. Point *C* will move only along the perpendicular bisector of the two points. Dragging point *C* will demonstrate that infinitely many different circles can be constructed through two points.



Passing through three given points — Constructing the perpendicular bisector between any two pairs of points produces lines that intersect at *C*, the center point of the circle. At least two perpendicular bisectors are needed to locate the center. When the second perpendicular bisector is added, the intersection of the two lines is the one point that is on both lines at the same time. Point *C* will not move since it is dependent on the position of the three points. This is true because point *C* is the intersection of two lines controlled by the position of the points.

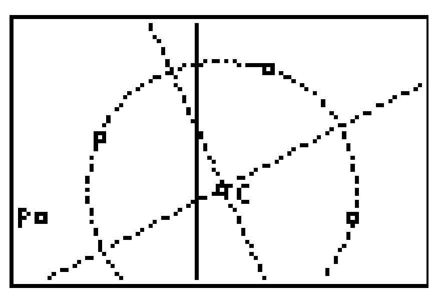


If the three points are collinear, then the perpendicular bisectors are parallel and do not intersect, thus no circle is formed because there is no center point.



Passing through four given points — When four points are drawn in the plane, one circle will pass through all four of the points when the perpendicular bisectors between all the representative points are coincident at a point. This point is the center of the circle and is equidistant from all four points.

If a circle passes through four points, then these points define a cyclic quadrilateral (see Activity 24 for more information on cyclic quadrilaterals.) Any set of non-collinear points will lie on a circle only when the perpendicular bisectors between the points are coincident at a point.



As in the case of three points, the four points cannot be collinear. Additionally, it is possible to select four non-collinear points that do not have a circle that will pass through every point.

2. How many points are needed to determine a unique point that can be used as a center of a circle that passes through all of the given points? Explain your reasoning and be prepared to demonstrate.

   In general, it takes just three non-collinear points to determine a unique circle since the center of this circle is fixed by the intersection of two of the three possible perpendicular bisectors between the three points. Adding more points does not change the location of the center of the circle through three points. One or two points cannot determine a unique circle because there are not enough points to fix the center of the circle.