

The Spread of Disease with Differential Equations

by

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Textbook Correlation: Key Topic

- Differential Equations

NCTM Principles and Standards:

- Process Standard
 - Representation
 - Connections
 - Problem Solving

After winter break a student infected with a flu virus returns to your dorm. One hundred students live in the dorm. The pattern of the spread of disease in seven days is illustrated in the table below.

Number of Days	Number of Infected Students
1	1
2	2
3	4
4	8
5	13
6	24
7	36

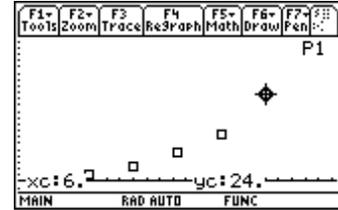
Exercise 1:

Enter the data in the Data Matrix Editor of a TI-89 or TI-92 Plus. Draw a scatter plot.

Solution:

F1 Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA	days	no. inf				
	c1	c2	c3			
1	1	1				
2	2	2				
3	3	4				
4	4	8				
r1c1=1						
MAIN	RAD	AUTO	FUNC			

F1 Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA	days	no. inf				
	c1	c2	c3			
4	4	8				
5	5	13				
6	6	24				
7	7	36				
r7c1=7						
MAIN	RAD	AUTO	FUNC			



Exercise 2:

Use the Logistic Population Model to analyze the spread of disease.

Solution:

The logistic population model states that the rate of change of the infected population with respect to time is directly proportional to the product of the number of people infected times the number of people who are not infected. This statement is represented by the differential equation $y' = ky(100-y)$ and initial condition $y(1) = 1$ where

y' = the rate of change of the number of infected people with respect to time;

y = the number of infected people;

t = time in seconds; and

k = the constant of proportionality.

- a) Use the command **deSolve** on the Home screen to solve the differential equation.

$y' = k \cdot y \cdot (100 - y)$
 $y(0) = 1$
 $y = \frac{100 \cdot e^{100 \cdot k \cdot t}}{e^{100 \cdot k \cdot t} + 99}$

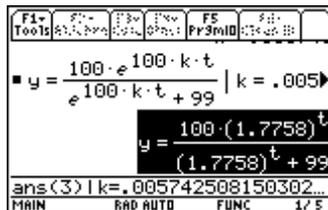
- b) Use another data point to obtain an equation with one variable, k .

$y = \frac{100 \cdot e^{100 \cdot k \cdot t}}{e^{100 \cdot k \cdot t} + 99} \quad | \quad t = 7 \text{ and } y = 36$
 $36 = \frac{100 \cdot e^{700 \cdot k}}{e^{700 \cdot k} + 99}$

- c) Solve for k .

$36 = \frac{100 \cdot e^{700 \cdot k}}{e^{700 \cdot k} + 99}$
 $\text{approx}(\text{solve}(36 = \frac{100 \cdot e^{700 \cdot k}}{e^{700 \cdot k} + 99}, k))$
 $k = 0.005743$

d) Substitute this value for k in the solution to the differential equation.

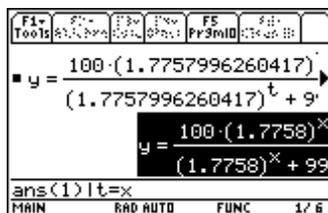


Answer: The mathematical model is highlighted in the screen above.

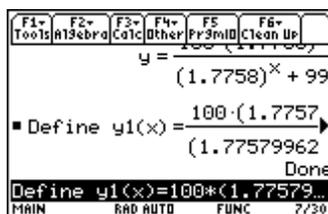
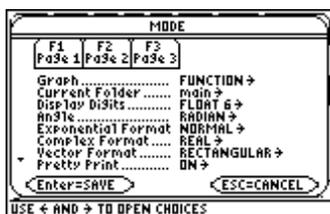
Exercise 3:

Solution:

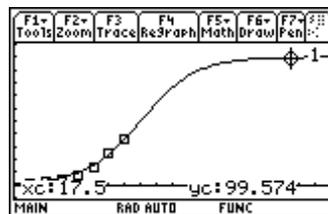
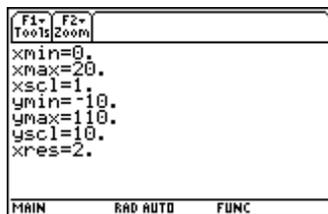
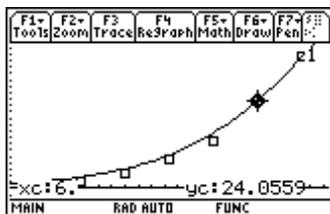
1. Substitute x for t so that you can enter the solution of the differential equation in **Function MODE**.



2. Copy this equation into the Y= editor. Make sure the TI-89 is in **FUNCTION MODE**.



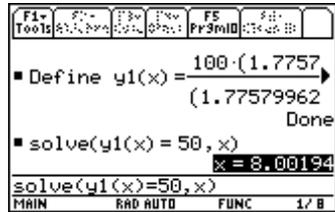
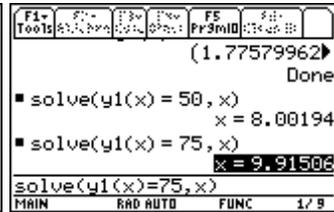
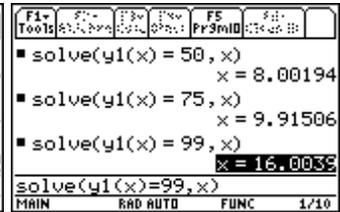
3. Use **F2 Zoom, 9:ZoomData** to graph the scatter plot of the data and the solution of the differential equation, y1(x) in this case. Change the window.



Exercise 4:

Use the model, $y_1(x)$, to predict when the number of infected students will be 50, 75, and 99.

Solution:

 <pre>F1- Tools Pr3mID Done Define y1(x) = 100 * (1.7757) / (1.77579962) Done solve(y1(x) = 50, x) x = 8.00194 solve(y1(x) = 50, x)</pre>	 <pre>(1.77579962) Done solve(y1(x) = 50, x) x = 8.00194 solve(y1(x) = 75, x) x = 9.91506 solve(y1(x) = 75, x)</pre>	 <pre>solve(y1(x) = 50, x) x = 8.00194 solve(y1(x) = 75, x) x = 9.91506 solve(y1(x) = 99, x) x = 16.0035 solve(y1(x) = 99, x)</pre>
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Answer: The model, $y_1(x)$, predicts that
50 students will be infected with the flu after 8 days..
75 students will be infected with the flu after 9.9 days..
99 students will be infected with the flu after 16 days..

Exercise 5:

Compare the model with the data points.