NUMB3RS Activity: Where’s Cheetah? Episode: “Finders Keepers”

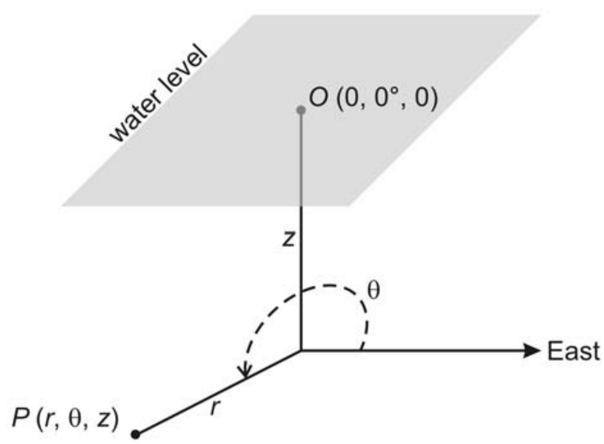
Topic: Simulation, Cylindrical Coordinates **Grade Level** 10 - 12
Objective: Predict a reasonable search area for a sunken yacht, based on several simulations
Time: 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

The Cheetah, a world-class racing yacht, sinks en route to the United States. The FBI suspects this was more than just an accident, and Charlie is enlisted to help locate the sunken vessel. The NSA (National Security Agency) has the exact location where the yacht's homing beacon shut down–presumably when the boat sank. However, the yacht is not at the expected location. Charlie is perplexed until he sees a leaf falling to the ground, wafting lazily back and forth in the breeze. He has a flash of insight. The NSA is assuming the boat sank like a stone, straight to the bottom. But because the boat was designed to glide through the water, it may have drifted as it sank. As Charlie puts it, “A boat like the Cheetah, a boat of that sophisticated a design… It's practically going to fly underwater, traveling laterally 4, maybe 5, feet for every foot it falls.” This activity simulates several ways the yacht could have sunk and how to determine a search radius to find it.

Discuss with Students

Cylindrical coordinates will be used to describe the location of the Cheetah, with (0, 0°, 0) representing the location where the boat sank. For the location $P(r, \theta, z)$, z is the distance from the surface of the water, θ is the angle of rotation counterclockwise, and r is the perpendicular distance from P to the z-axis. The distance z is measured along the z-axis, which is perpendicular to the plane of the surface of the water at (0, 0°, 0). The angle θ represents the direction which the boat is pointing (θ = 0° being due east, θ = 90° due north, etc.). Assume that once the boat begins to sink and veer in a particular direction, it will continue to veer in that direction.



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All students will get the same answer for Question 1, where the boat is assumed to travel 4 feet horizontally for every 1 foot it descends at an angle of 225°. In Questions 2–4, students use the answer key iteration feature of their calculators, randomly choosing the angle the boat will veer using the command: **{0, randInt(0,359), 0} → {*r*, *θ*, *z*}**. In successive steps, students will introduce a random distance between 30 feet and 60 feet that the boat moves horizontally for each 10 feet it descends by entering **{ANS(1) + randInt(30,60), ANS(2), ANS(3) – 10}**. Students will then repeatedly press [ENTER] until the third coordinate is the desired depth (–200, to represent a depth of 200 feet in Question 2). The search region is an annulus (washer) whose inner radius is (30 × depth) / 10 and whose outer radius is the largest value of *r* obtained in the simulations. When the students repeat the simulation starting from the assignment of a random angle, the final values of *r* may vary, but choosing the largest value is a reasonable choice for the outer radius of the annulus.

Student Page Answers:
1a. *(800, 225°, –200)* **1b.** *800 feet* **2.** *Answers will vary but (942, 302°, –200) is the result for the example shown.* **3a.** *Answers will vary but will be similar to (875, 84°, –200)* **3b.** *Answers will vary, but an annulus with inner radius of 20 × 30 = 600 feet and an outer radius of the largest value of r obtained in the six trials is reasonable.* **4a.** *Answers will vary but will be similar to (1275, 104°, –300)* **4b.** *Answers will vary, but an annulus with inner radius of 30 × 30 = 900 feet and an outer radius of the largest value of r obtained in the six trials is reasonable.* **4c.** *If r_{200} is the maximum value of r obtained in 3b and r_{300} is the maximum value of r obtained in 4b, then $(\pi r_{300}^2 - 900^2\pi) - (\pi r_{200}^2 - 600^2\pi)$ is reasonable.*

Name: ________________________________ Date: _______________

NUMB3RS Activity: Where's Cheetah?

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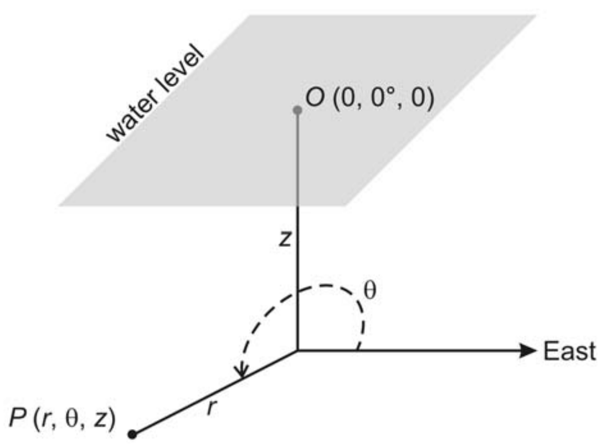
Cylindrical coordinates will be used to describe the location of the Cheetah, with (0, 0°, 0) representing the location where the boat sank. For the location $P(r, \theta, z)$, z is the distance from the surface of the water measured along the z-axis perpendicular to the plane of the surface of the water at (0, 0°, 0); θ is the angle of rotation representing the direction which the boat is pointing, with $\theta = 0°$ representing due east, $\theta = 90°$ due north, etc.; and r is the perpendicular distance from P to the z-axis.



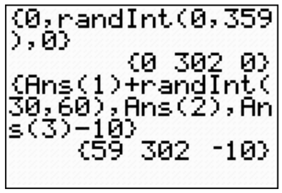
1. Suppose the Cheetah is pointing in a southwest direction and drifts horizontally 4 feet for every 1 foot it descends.

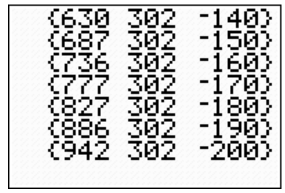
a. Suppose the Cheetah sinks to a depth of 200 feet. Find the coordinates (r, θ, z) of its final location.

b. Suppose you do not know the direction in which the boat pointed. Find an appropriate search radius that divers could use to locate the Cheetah.

2. Now, use your calculator to generate a random value for the fixed angle that the Cheetah veers as it sinks. Assume that the boat randomly drifts between 30 feet and 60 feet horizontally for every 10 feet it descends.

Carry out your simulation using answer key iteration on your calculator as shown in the screen on the left below. [Note: to insert the **randInt(** command, press [MATH], go to the **PRB** menu, and select **5:randInt(**.] Repeatedly press [ENTER] until the third coordinate is –200. Remember, the answers will be in the form {*r*, θ, *z*}.





If the boat sinks to a depth of 200 feet, give the coordinates (*r*, θ, *z*) of its final location. (In the example shown, the boat sank at an angle of θ = 302° and reached a distance of *r* = 942 feet from where it started at a depth of 200 feet (*z* = –200).)

3. **a.** To determine a search region, repeat the process from Question 2 five more times to determine five more possible locations of the Cheetah. (Note: To save time, use [2nd] [ENTRY] to display the last entered command. Using this key more than once will recall previously entered commands.)

b. Using the locations you found in part a, find an appropriate search region in which the divers should look for the Cheetah. (Hint: give a maximum radius and a minimum radius.)

4. Suppose the Cheetah sank to a depth of 300 feet (*z* = –300).

a. Carry out the process from Question 2 five times to determine five possible locations for the Cheetah.

b. Using the locations you found in part a, find an appropriate search region in which the divers should look for the Cheetah.

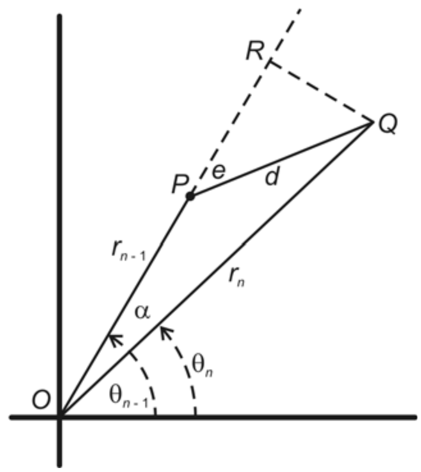
c. How much larger is the area of your search region in Question 4b than the one you found in Question 3b?

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

1. Suppose the Cheetah changes the direction in which it veers every 10 feet as it descends through the water. Assume this amount is a fixed random number, *e*, between –15° and 15°. Also assume it randomly drifts horizontally a fixed random distance *d* between 30 feet and 60 feet for every 10 feet it descends. This descent is equivalent to the boat descending 10 feet immediately and then moving laterally *d* feet, so the change can be modeled in a plane parallel to the ocean's surface as shown in the diagram below. For finding the angle θ, the angle *e* is positive if the boat turns to the left from the direction it was going (and negative if it turns to the right). In the diagram below, $e < 0$.



This process can be modeled using the equations below.

(1) $r_n^2 = r_{n-1}^2 + d^2 - 2r_{n-1}d\cos(180 - e)$, $r_1 = d$

(2) $\theta_n = \theta_{n-1} + \tan^{-1}\left(\dfrac{d\sin e}{r_{n-1} + d\cos e}\right)$, $\theta_0 = \text{randInt}(0, 359)$

(3) $z_n = z_{n-1} - 10$, $z_0 = 0$

a. Using the Law of Cosines, explain why equation (1) is valid.

b. Explain why equation (2) is valid. (Hint: look at the auxiliary segments drawn in the figure or use the Law of Sines and possibly the formula for sin ($a + b$).)

Note: for the case in the diagram, $e < 0$ for finding θ, but $e > 0$ for finding r_n. Recalling that cos e = cos($-e$) and cos(180 + e) = cos(180 – e) shows the results obtained in equations (1) and (2) will be correct.

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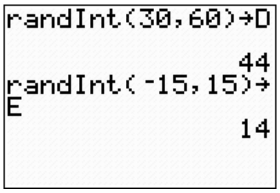
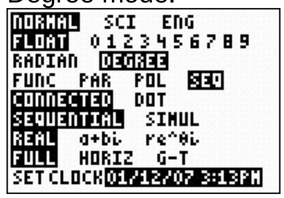
c. On the home screen of your calculator, enter **randInt(30, 60) → D** and **randInt(–15, 15) → E** to create a random drift distance and a random direction, respectively, for equations (1), (2), and (3). Then enter these equations while in sequence mode, letting $u(n) = r_n$, $v(n) = \theta_n$, and $w(n) = z_n$.

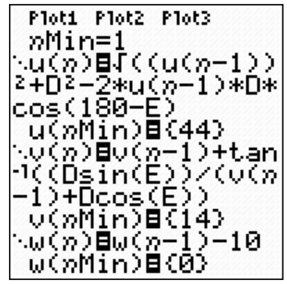
Store random integers as D and E.



Set your calculator to Sequence mode and Degree mode.



Enter the equations in $u(n)$, $v(n)$, and $w(n)$.



d. Use the table feature on your calculator to run at least three simulations of the descent of the Cheetah. Use different values of *d* and *e* each time. If the boat sinks to a depth of 200 feet, give the coordinates (r, θ, z) of its location each time.

e. Using these locations, find an appropriate search region in which the divers should look for the Cheetah.

2. Write a program to simulate the process in Question 1 above, but allow the small angle the boat veers to vary randomly between –15° and 15° and the amount the boat moves horizontally to vary randomly between 30 and 60 feet for every 10 feet it descends.

Additional Information

For more information on cylindrical coordinates, consult the following Web sites:

- **http://mathworld.wolfram.com/CylindricalCoordinates.html**
- **http://www.math.montana.edu/frankw/ccp/multiworld/multipleIVP/cylindrical/body.htm**

For more information on "search theory" with relation to underwater search, visit the U.S. Coast Guard Search and Rescue site: **http://www.uscg.mil/hq/g-o/g-opr/g-opr.htm**

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