

## NUMB3RS Activity: Expected Value Episode: "Longshot"

**Topic:** Geometric probability, expected value

**Grade Level:** 9 - 10

**Objective:** In this activity, students will use geometric probability and expected value to analyze a land claim scenario.

**Time:** 15 - 20 minutes

**Materials:** TI-83 Plus/TI-84 Plus graphing calculators, TI-Navigator™ system, and the following activity settings files: *LandClaim1.act*, *LandClaim2.act*, and *LandClaim3.act*. To download these files, go to <http://education.ti.com/exchange> and search for "7915."

### Introduction

In "Longshot," a man by the name of Danny Roberts is murdered at a horse racetrack moments after he wins his bet on a Pick 6 race. A notebook filled with horse-racing data and equations is found on the victim and brought to Charlie for his analysis. Charlie's analysis reveals a couple of interesting things. First, he finds that Roberts' equations were designed to pick the second-place finisher. Charlie's theory is that Roberts was looking for bargains. As Charlie explains, "The racetrack uses a pari-mutuel betting system. Market forces determine the odds. In pari-mutuel betting, the odds fluctuate according to how much is wagered on each horse. It's like an auction. The more people who want to buy something, the more expensive it becomes." By picking a second-place finisher, Roberts hopes to "cash in" on a horse for which the betting has been relatively light.

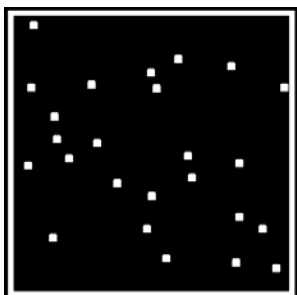
Charlie also discovers that, over time, Roberts modified his approach by making certain adjustments to his equations. Charlie speculates that Roberts made these changes because the race outcomes were consistently falling outside the range of realistic probabilities. In short, Roberts retooled his system to account for the fact that the races were fixed. Unfortunately, the victim's knowledge of this apparent scheme likely led to his untimely death.

A pari-mutuel betting system is often used in horse racing and other sporting events of relatively short duration. With this type of system, payoff amounts are not determined until the betting has been closed and winners simply receive a share of the entire pool, less what goes to the gaming facility (the "house"). From the standpoint of the racetrack or gaming facility, there is no risk at losing money in any given event. The only variable has to do with the actual volume of betting and where the bets are placed. Conversely, fixed-odds betting facilities, such as casinos, declare payoff amounts before bets are placed; however, they routinely set these odds so there is a slight "house advantage." While casinos typically experience short-term losses (for example, a player might "hit the jackpot"), they rely on long-term, high volume betting to obtain an overall profit. Fixed odds betting can be analyzed via the concept of expected values.

In this activity, students will study concepts of pari-mutuel betting and expected values using the context of a situation involving land claims. This will be accomplished by having students make speculative investments in land parcels that will be divided up among all students. Students will then calculate their losses or gains using geometric probability and betting concepts similar to those found in the *Longshot* episode.

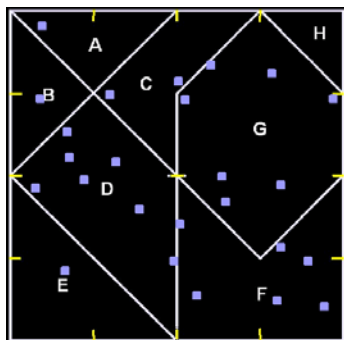
### **Part I: Claiming Land**

1.
  - a. Launch TI-Navigator™ on the computer and press **Begin Class** to start the session.
  - b. Have students log into NavNet on their calculators.
  
2.
  - a. Load the **LandClaim1.act** activity settings file into Activity Center, and click the 'Graph' tab. This will display a square in the Activity Center window.
  - b. Ask students to imagine they have been invited to invest \$50,000 to become co-owners of one of eight parcels of government-owned land contained within the square shown in Activity Center. However, the parcels have many different shapes and sizes (these will not be revealed for students to see until later in the activity). The goal of the students is to mark a point somewhere within the square, with the hope that they will have marked a point in a parcel whose value (due to its size and/or number of marked points) is greater than their \$50,000 bid.
  - c. Explain to students that the collective amount bid will be considered a fair amount for all 8 land parcels combined. For example, a class of 25 students bidding \$50,000 per student will collectively bid \$1,250,000 for the entire land area. However, students should recognize that each section of land area, when divided, may be worth different amounts based on the area they occupy. Also mention to students that if the land is found to be worth more than the initial investment, students will divide the profit; if the land is worth less than the original investment, students will divide the loss. (Note: These scenarios are considered in the extension to this activity.)
  - d. Press **Start Activity** and tell students to move their cursors to any point they wish within the square and mark a point.
  - e. Press **Stop Activity** after all the students have marked their points. A sample of students' points is shown below.



### **Part II: Calculate Expected Value**

3.
  - a. Load the **LandClaim2.act** activity settings file into Activity Center to reveal the land parcels. The marked points will be preserved as long as you do not clear the activity data.

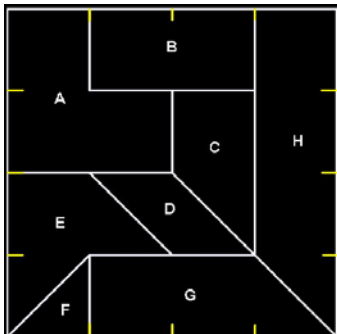


Note: The entire square can be broken into 32 regions, each equal in the size and shape to Region *H*. For example, it can be seen that region *F* comprises of 6 of these units (the yellow tick marks are intended to help students identify the number of units in each area). Therefore, a randomly placed point has a probability of  $\frac{6}{32} = \frac{3}{16}$  of landing in Region *F*.

- b. Ask students to look at where their points landed and discuss with them whether they think their claim was fair. Using **Quick Poll** (with *Open Response*), ask students to calculate the value a student's net gain or loss for a specific region. In the example above, the one student that landed in Region *E* paid \$50,000 for the land, which has a total value of  $\frac{4}{32} \cdot \$1,250,000 = \$156,250$ , giving him a profit of  $\$156,250 - \$50,000 = \$106,250$ . The screen at left below shows a response with the *Numeric* box checked, giving the value of the calculation. The screen at right shows the same response with the *Numeric* box unchecked, showing the actual calculations made by the student. Use this feature to discuss how the probabilities and expected values are calculated.



- c. Using **Quick Poll** (with *Open Response*), have each student now calculate the net profit or loss for the region for which they marked. For example, each student that landed in Region *D* had a net loss of  $\frac{6}{32} \cdot \frac{\$1,250,000}{7} - \$50,000 \approx -\$16,517.86$ .
- d. Finally, using **Quick Poll** (with *Open Response*), ask students what they think the sum of the net gains and losses should equal. Students should recognize that the result is zero since the contributions from each student are entirely given back in terms of value of land. Explain to students this is a form of betting system in which the governing authority's advantage is zero (since the entire land area is given back to the students). This is similar to a pari-mutuel betting system in which the governing authority (the "house") assumes no risk – the only unknown is the distribution of land at the conclusion of the bidding process.
- e. This activity can be repeated with a different land arrangement by clearing the activity data and starting again with Part I. In Part II, load the **LandClaim3.act** activity settings file into Activity Center to reveal the land parcels shown below. All other steps can be followed in the same way.



*The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.*

## Extensions

- This land area scenario can be modified such that the outcome is not fair for the bidders. For example, consider a situation in which it is determined that one of the regions in the original square is deemed worthless, perhaps due to the discovery of toxic contaminants. The government (who collects this money) will receive money for a region that they know is worthless. This is analogous to horse racing (and other pari-mutuel betting schemes) in which the racetrack takes a percent of the bets off the top. Furthermore, assume that as part of the original contract, each bidder agrees to split such losses evenly amongst themselves, rather than being compensated by the government.
- Tell students that toxic contaminants were discovered in one of the regions (such as Region *H*), deeming the region worthless. Have students calculate the expected value of the remaining regions, using **Quick Poll** (with *Open Response*) to collect responses. For example, the expected value of a \$1 bid in Region *F* becomes  $(-1 \times \frac{26}{32}) + (\frac{25}{6} \times \frac{6}{32}) = -\frac{1}{32}$ . This can be translated to a loss of  $\$50,000 \cdot \frac{1}{32} = \$1,562.50$ .
- You can also analyze a situation in which people bidding on the land gain an advantage. Consider a situation where precious minerals such as gold, silver, and coal are discovered on select land parcels. Assign an additional value to these regions and use **Quick Poll** (with *Open Response*) to collect these newly computed expected values.

## Additional Resources

- For more information on *expected values* see <http://planetmath.org/?op=getobj&from=objects&id=505>.
- Additional information on *pari-mutuel* betting can be found at the following Web sites:  
<http://www.encyclopedia.com/doc/1E1-parimutu.html>  
<http://casinogambling.about.com/od/racesports/a/Parimutuel.htm>
- The *NUMB3RS* activity "Not So Great Expectations" further explores the pari-mutuel betting system. To download this activity, go to <http://education.ti.com/exchange>, and search for "7599."
- If you would like to learn more about TI-Navigator™, visit <http://education.ti.com/navigator>.