

Name _____

Date _____

EXPLORATIONS

ACTIVITY 6

Warming Up to Heating Curves

If you take a cold drink from your refrigerator and place it on the counter, it will start to warm up. The warming process is quick at first, and then slowly levels off as the beverage approaches room temperature. In this situation, the rate at which the temperature of the drink changes is approximately proportional to the difference between its temperature and the temperature of its surroundings. Although this rule is commonly called *Newton's Law of Cooling*, the principle applies to warming as well as cooling phenomena.

In this activity, you will use the Calculator-Based Laboratory 2™ System (CBL 2™) to collect temperature data as a sensor warms up in a cup of hot water. Then you will use the Voyage™ 200 PLT with DataMate software to find an appropriate mathematical model for the resulting data set.

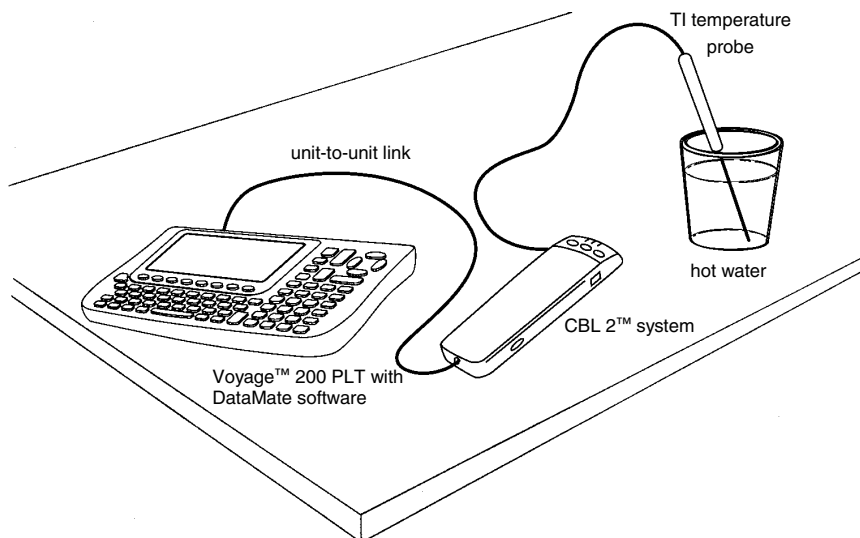
Instructions

Part A—Setup and Data Collection

Setup

You need:

- 1 CBL 2 unit with unit-to-unit linking cord
- 1 Voyage 200 PLT with DataMate software
- 1 TI temperature probe
- 1 cup of hot water



Warming Up to Heating Curves *(Continued)*

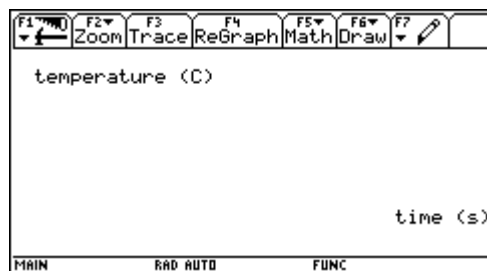
Data Collection

In this activity, you will record temperature variations, in degrees Celsius, as a temperature probe is heated in a cup of hot water. The cup of hot water to be used in the experiment should be very hot, but does not need to be boiling.

1. Connect the Voyage™ 200 PLT and CBL 2™ system using the unit-to-unit linking cord. Push firmly on each end of the cord to be sure the connection to each unit is good.
2. Connect the TI temperature probe into the Channel 1 (~CH1) port on the CBL 2 system. *Do not place the probe in the water at this time.*
3. Start DataMate by pressing [APPS], selecting DataMate, and pressing [ENTER].
4. Press [CLEAR] to reset DataMate.
5. When you are ready to start collecting data, simultaneously place the temperature probe in the hot water and press [2] on your Voyage 200 PLT. A graph of the temperature data you are collecting will be displayed.
6. When the temperature appears to stop rising, press [STO▶] to end the data collection. Data collection should last approximately 30 seconds.
7. Your data should show temperature values (in degrees Celsius) that increase rapidly at first and then level off as time passes.

If your data set is not satisfactory, press [ESC] and repeat steps 5 and 6 to do another trial. *Be sure to allow the probe to cool sufficiently before collecting a new set of data.*

If your data set is satisfactory, sketch a plot of your temperature versus time data in the space provided to the right. Be sure to include axes on your sketch.



Warming Up to Heating Curves *(Continued)*

Part B—Data Analysis

1. The temperature data you collected can be modeled using a modified exponential function of the form

$$y = a(1 - b^x) + c$$

where x is time and y is temperature. Note that when $x = 0$, the equation reduces to $y = c$, where c is the y -intercept.

Press \blacktriangleleft and \blacktriangleright to trace along the plot. Identify the starting temperature, and record it as c in the space below.

$c =$ _____

2. Trace along the plot and determine the highest temperature reached. Round this temperature up to the next whole degree (as in $79.24 \rightarrow 80$), and record this y -value below as the approach temperature.

approach temperature = _____

You can now exit DataMate. To do this, press $\boxed{\text{ESC}}$ to return to the main screen and then press $\boxed{6}$ to quit. Press \blacklozenge $\boxed{\text{GRAPH}}$ to again view the graph. If necessary, press $\boxed{\text{F2}}$ $\boxed{9}$ to rescale the graph based on your data.

Warming Up to Heating Curves *(Continued)*

3. As time increases without bound in our heating model, b^x becomes closer and closer to zero, and the temperatures approach the value $a + c$. Knowing the value of c from step 1 and the approach temperature from step 2, solve for the value of a and record it below.

$a =$ _____

4. Press \blacklozenge [Y=] to display the Y= Editor. To clear any existing functions, press $\boxed{F1}$, select 8:Clear Functions, and press \boxed{ENTER} .

Press \boxed{ENTER} to move the cursor to the entry line.

Press \mathbf{A} $\boxed{\times}$ $\boxed{(}$ $\mathbf{1}$ $\boxed{-}$ \mathbf{B} $\boxed{\wedge}$ \mathbf{X} $\boxed{)}$ $\boxed{+}$ \mathbf{C} \boxed{ENTER} to store the expression as function $y_1(x)$.

Press \blacklozenge [CALC HOME] to return to the Home screen.

At the entry line, type the value of a found in step 3, and then press $\boxed{STO\blacktriangleright}$ \mathbf{A} \boxed{ENTER} to store the value of the variable a on the Voyage™ 200 PLT.

Type the value of c found in step 1, and then press $\boxed{STO\blacktriangleright}$ \mathbf{C} \boxed{ENTER} to store this value to c .

5. To obtain a curve that fits the data well, you will need to find the value of b by trial and error. Start by storing 0.5 to the variable b as described above. Press \blacklozenge [GRAPH] to display the data and the modeling curve on the same screen.

Try to improve the fit by storing different numbers for the variable b . View the graph for each new value of b that you test. Experiment until you find a value that provides a good fit for the data.

Record in the space below the b value that works best.

$b =$ _____

Warming Up to Heating Curves (Continued)

6. An alternate approach for finding a mathematical model for the temperature data involves using the exponential regression feature on the Voyage™ 200 PLT with a few modifications. The Voyage 200 PLT models exponential data with an equation of the form $y = ab^x$. The model you used earlier was of the form

$$\begin{aligned}y &= a(1 - b^x) + c \\ &= a - ab^x + c \\ &= -ab^x + (a + c).\end{aligned}$$

Notice that the final form of this equation can be transformed into the form $y = ab^x$ by subtracting the sum $a + c$, and then flipping the data over the x -axis.

In mathematical terms, you need to apply a *shift* and a *reflection* to the temperature data to use the Voyage 200 PLT model.

To accomplish a shift and a reflection, press $\boxed{\text{APPS}}$, select Data/Matrix Editor, press $\boxed{\text{ENTER}}$, and then select 1:Current. The temperature data you collected with the CBL 2™ is shown in the Data/Matrix Editor.

Move the cursor to any cell in column c3, and press $\boxed{\text{F4}}$. Define c3 by pressing $\mathbf{A} \boxed{+} \mathbf{C} \boxed{-} \mathbf{C2} \boxed{\text{ENTER}}$.

All of the modified temperature values appearing in column c3 have been adjusted by the desired shift and reflection.

7. To perform an exponential regression on the data in columns c1 and c3, press $\boxed{\text{F5}}$ \downarrow and select 4:ExpReg.

Press \downarrow and enter **C1** for the x . Press \downarrow and enter **C3** for y . Press $\downarrow \rightarrow \downarrow \downarrow \boxed{\text{ENTER}}$ to store the regression equation to $y2(x)$.

Press $\boxed{\text{ENTER}}$ again to perform the calculation and display the exponential regression equation. Record in the spaces below the ExpReg values that appear in the dialog box.

$a =$

$b =$

Are the regression values of a and b consistent with your results from step 3 and step 5?

Warming Up to Heating Curves *(Continued)*

8. Press \blacklozenge [Y=], and move the cursor to highlight equation $y_2(x)$. Press $\boxed{\text{ENTER}}$ to copy $y_2(x)$ to the entry line and \blacklozenge to position the cursor at the beginning of the expression.

Press \mathbf{A} $\boxed{+}$ \mathbf{C} $\boxed{-}$ $\boxed{\text{ENTER}}$ to adjust for the shift and reflection you made earlier.

Highlight equation $y_2(x)$ again. Press $\boxed{\text{F6}}$ and select 4:Thick to set the graphing style to a thick line so the regression curve will be distinguishable from the previous modeling curve.

Press \blacklozenge [GRAPH] to see the data and the two curves. How well does each equation fit the temperature data?

Warming Up to Heating Curves *(Continued)*

Questions

Use the results of the activity to answer the following.

1. Use your own words to describe how the values of a and c affect the temperature-versus-time model, $y = a(1 - b^x) + c$.

2. Explain why the value of b must be less than 1 but greater than 0. How would the shape of the graph be affected if b were greater than 1?
