

How Do You Beat the System? Teacher Notes

Many students have great difficulty accurately performing the number of sequential steps required to find the solutions to systems of equations by those methods typically taught in Algebra I. Using matrices and the multiplication with inverse matrices is a more manageable task for many students because the computations are performed by the calculator. In standardized testing setting, there is rarely a question asked for which there is no inverse of a matrix indicating that there is no solution.

How can you beat the “system” of equations? Is there a way to use the graphing calculator to solve a system of equations other than graphing?

Let’s look at a way to use the graphing calculator that may be different than what you have seen before. Look at the equation given below.

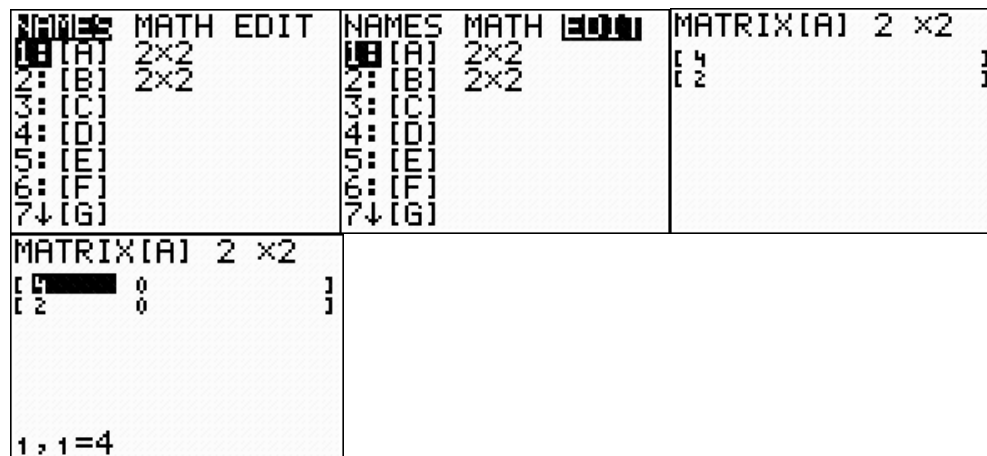
$$2x - 3y = 5$$

$$x + 2y = -1$$

Notice that these equations are not in the form that you usually see when you use the calculator where the $y =$ to the rest of the terms.

You can use matrices to solve this system of equations. Here is how you enter the information in the graphing calculator.

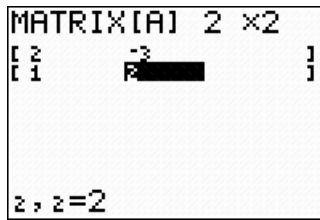
The multipliers or coefficients for the variables can be put into a matrix. To create the matrix, press $\text{2nd} \text{ [x}^{-1}\text{] } \text{[2]} \text{ [2]}$ to edit a matrix named A. Create the matrix to have 2 rows by 2 columns.



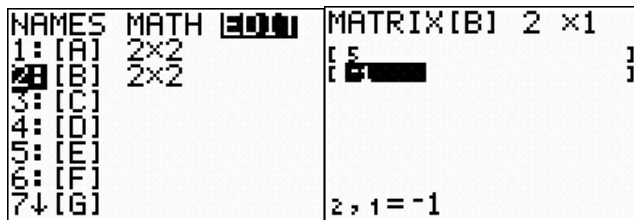
Enter the coefficients for x and y from the first equation into row 1 and the coefficients for x and y in the second equation in row 2 as shown below.

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Date: _____



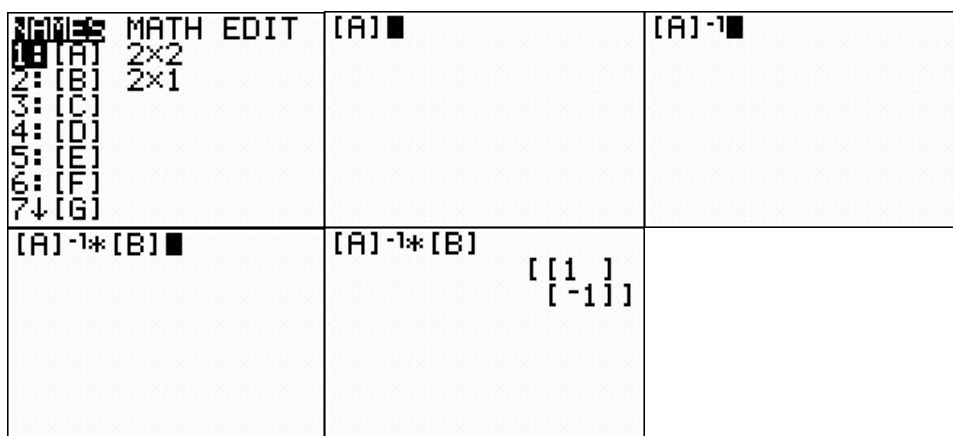
Create a matrix B that has 2 rows and 1 column into which you will enter the constants.



Instead of adding the equations together or looking at the graphs, you can use the matrices you have created to find the solution for the system of equations. You can multiply the inverse of matrix A (from the coefficients) by matrix B (the constant values to which the expressions are equal) and you will get the values for x and y that make both equations true.

To stop editing a matrix and to begin performing operations with the matrices, enter $\boxed{2\text{nd}}\boxed{\text{MODE}}$ to **quit** the previous task.

Enter $\boxed{2\text{nd}}\boxed{\text{MODE}}\boxed{\text{ENTER}}$ to recall the name of Matrix A. It will be copied to the homescreen. To select the inverse of A, press $\boxed{x^{-1}}$ and, recalling the name of the matrix as before, multiply by matrix B. Press $\boxed{\text{ENTER}}$ and you will see a matrix with 2 rows and 1 column.



The number in the top row, 1, is the value of x and the number in the second row, -1, is the value of y. The solution could be written (1, -1) to represent the point at which the graphs of the two equations would intersect.

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Follow the same steps to solve the system below. Record your matrices as you find the solution.

$$\begin{aligned}x - y &= -3 \\ 3x + y &= -1\end{aligned}$$

$$A = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$B = \begin{bmatrix} \\ \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} \\ \end{bmatrix} \quad \text{Solution (,)}$$

The solution is (-1, 2)

What do you do if the equations are not in the same order as in the example below?

45 The equations of two lines are $6x - y = 4$ and $y = 4x + 2$. What is the value of x in the solution for this system of equations?

A $x = 14$

B $x = 3$

C $x = 1$

D $x = 6$

First, change the second equation to be in the same order as the first.

What would the new version of the equation look like? _____

$-4x + y = 2$

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Let's look at the two equations.

Use the step you have learned to solve the system of equations. Record the matrices you use and the solution you find.

$$A = \begin{bmatrix} & \\ & \end{bmatrix} \qquad B = \begin{bmatrix} \\ \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} \\ \end{bmatrix} \qquad \text{Solution (,)}$$

The solution is (3, 14).

Let's look at some other kinds of problems you can see.

$$-x + 2y = -2$$

$$Y = (1/2)x + 3$$

What is the form of the second equation you need to use a matrix? _____
-1/2 x + y = 3

Enter the matrices as shown below.

MATRIX[A] 2 × 2 [-1 2 [-1/2 1	MATRIX[B] 2 × 1 [-2 [3
Z, Z=1	Z, 1=3

See what happens when you multiply the inverse of matrix A by matrix B.

[A] ⁻¹ *[B] [[1] [-1]]	ERR: SINGULAR MAT Quit Goto
[A] ⁻¹ *[B] █	

What does this mean? It means that there is not a single point or ordered pair that represents the solution to the system of equations. These two equations represent lines that are parallel and there is no solution to this system of equations. Other times when you get this error message on the calculator, it

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means that the two equations represent the same line when graphed and they have an infinite number of points in common.

Can this method be used for more equations using more variables?

Consider the system of three equations in three variables.

$$2x+3y+z=-1$$

$$3x+3y+z=1$$

$$2x+4y+z=-2$$

Create matrix A with 3 rows and 3 columns.

$$A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Create matrix B with 3 rows and 1 column.

$$B = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$A^{-1} \times B = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$x=2 \quad y=-1 \quad z=-2$$

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Check for Understanding

Answer is H.

16 Which of the following is the solution for this system of linear equations?

$$y = -\frac{2}{3}x + 2$$

$$3x - y = -13$$

F $(\frac{17}{3}, 4)$

G $(-1, \frac{8}{3})$

H $(-3, 4)$

J $(-3, -4)$