



Math Objectives

- Students will identify the shape of a distribution as being skewed or mound-shaped and approximately symmetric.
- Students will recognize that a normal probability plot of skewed data is nonlinear and either concave up or concave down.
- Students will recognize that a normal probability plot of approximately normal data is approximately linear.
- Students will identify outliers on a normal probability plot.
- Use appropriate tools strategically (CCSS Mathematical Practices).
- Look for and make use of a pattern or structure (CCSS Mathematical Practices).

Vocabulary

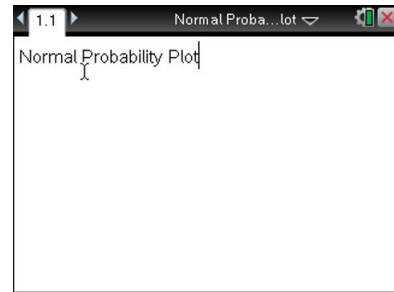
- normal
- normal probability plot
- outlier
- skew
- symmetric

Prerequisites

- Students should be familiar with z-scores and normal distributions.

About the Lesson

- This lesson involves creating a normal probability plot for several data sets involving height to examine the appearance of such plots when the distribution is approximately normal.
- As a result, students will:
 - Use a histogram to discuss the shape of the distribution of a data set.
 - Describe a normal probability plot for data sets whose distributions are skewed, approximately normal, or contain outliers.
 - Move data values in a dot plot to investigate how the shape of a distribution compares to the linearity of a normal probability plot.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing **ctrl** **G**.

Lesson Files:

Student Activity

Normal_Probability_Plot_Student.pdf

Normal_Probability_Plot_Student.doc

Normal_Probability_Plot_Create.doc

TI-Nspire document

Normal_Probability_Plot.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.



TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine the normal probability plots of different shaped distributions.
- Use Live Presenter to demonstrate.

Discussion Points and Possible Answers

Tech Tip: Students have the option of either using the create document for instructions on creating normal probability plots for the data or uploading the already completed .tns file.

The data below were taken from a random sample of high school males. Normal probability plots help determine whether a data set is approximately normal by plotting expected z-scores vs. the data value. You will investigate the shape of histograms and the association between the data values and the respective expected z-scores to determine whether the distribution of a given data set is approximately normal.

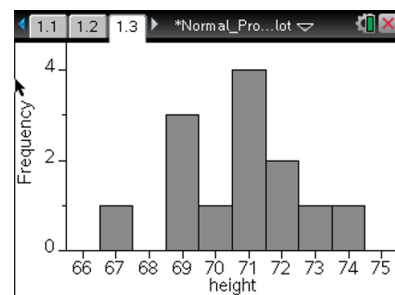
Survey data from random sample of males

Height (in)	73	74	71	67	69	71	72	70	69	72	71	69	71
Handspan (cm)	20.5	20.5	18.75	20	19.5	20.5	20.25	20.5	17	20.5	20.5	16	18
Shoe Size (cm)	32	30	31	29.5	31	29	39	32.5	31	32	30	30.5	31

Move to page 1.3.

Tip: Hovering over a bin in a histogram or a point in a normal probability plot will display the data.

1. a. Describe the distribution of the heights of the males. Explain your answer.



Sample Answers: The distribution of heights is approximately symmetric and mound-shaped. Most males are between 68.5 and 72.5 inches tall.

TI-Nspire Navigator Opportunity: Quick Poll

See Note 1 at the end of this lesson.



- b. Plot the mean for the male heights by selecting **[menu]** > **Analyze** > **Plot Value**. Use the alphabet keys to type **mean(height)**, and press **[enter]**. Now find the standard deviation by selecting **[menu]** > **Analyze** > **Plot Value**. Use the alphabet keys to type **stdevsamp(height)**, and press **[enter]**. Write down the mean and the standard deviation.

Sample Answers: The mean is about 71 inches, and the standard deviation is 1.89.

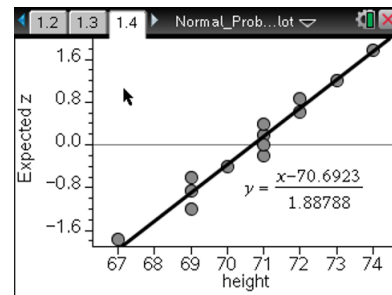
- c. Would you consider the distribution of male heights to be approximately normal based on the shape of the histogram and the mean and standard deviation? Explain your reasoning.

Sample Answers: The standard deviation is 1.89 inches. The distribution of male heights seems approximately normal because it is mound-shaped and approximately symmetric. The mean, at 71 inches, is near the center, and the data falls within three standard deviations on either side of the mean, between 66.5 and 74.5 inches.

Move to page 1.4.

Note: Clicking on the line will display the equation of the line.

2. a. The plot on Page 1.4 is called a normal probability plot. Move the cursor to one of the points. What do the coordinates represent?



Sample Answers: The point (69, -0.615) represents the student who is 69 inches tall, and the expected z-score for that height in a normal distribution would be -0.615.

- b. Describe the association between the height and the expected z-scores displayed in the graph.

Sample Answers: The normal probability plot is approximately linear in shape. The line seems to fit the data quite well.

- c. Based on your answer to question 1b, make a conjecture about the association seen in a normal probability plot and approximately normal data.

Sample Answers: Normal probability plots seem approximately linear when the data set is approximately normal.



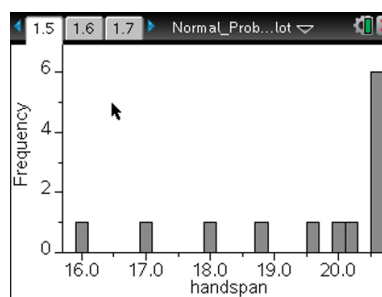
Teacher Tip: The line in the normal probability plot is defined by the

calculation of the z-scores, $y = \frac{x - \bar{x}}{s_x}$.

Teacher Tip: Discussion on the expected z-scores might be beneficial to students. Normal probability plots show the z-scores that should be expected if the data were approximately normal. The y-values of the plotted points do not change because they are expected z-scores; however, the variability in the x-values indicates a deviation from the expected.

Move to page 1.5.

3. Page 1.5 shows the distribution of a random sample of male handspans measured in centimeters.
 - a. Describe the distribution of the handspans. Explain your answer.



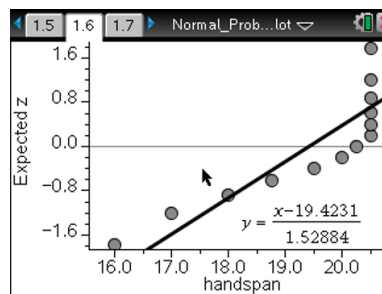
Sample Answers: The distribution of handspans is skewed left meaning most males have large handspans. In addition, there are gaps in the data.

- b. Based on the normal probability plot for heights, make a conjecture about the normal probability plot for the handspans. Do you think it would be similar or different? Explain your answer, and make a sketch.

Sample Answers: The normal probability plot appeared linear for the male heights, and the histogram was approximately normal. Since the histogram for handspans is skewed, I think the normal probability plot will be nonlinear.

Move to page 1.6.

4. Page 1.6 displays the normal probability plot for the handspans. Does the graph support your conjecture in 3b? Describe the graph.



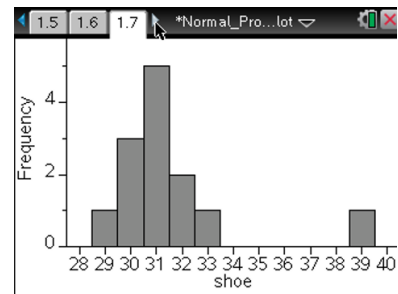
Sample Answers: The normal probability plot supports my conjecture because it is not linear. In fact, it appears to be curved and concave up. The data are above the line, then below the line, then above the line.



Move to page 1.7.

- Describe the distribution of the lengths of male shoe sizes in cm.

Sample Answers: The distribution of shoe sizes appears approximately symmetric and mound-shaped with most males having shoe sizes between 29.5 cm and 32.5 cm. However, there is an apparent outlier, 37 cm.



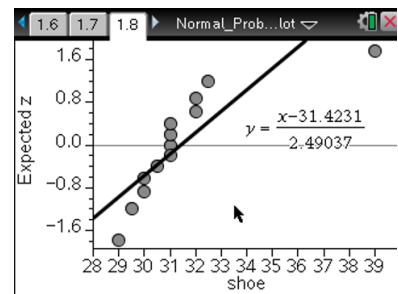
- Based on the normal probability plots for heights and handspans, make a conjecture about the normal probability plot for the shoe sizes compared to the normal probability plots for the heights and for the handspans. Explain your thinking, and make a sketch of what you think the plot will look like.

Sample Answers: Sketches will vary. The normal probability plot appeared linear for the male heights, which was also symmetric and mound-shaped, so I think the normal probability plot for shoe sizes should also look approximately linear. However, the shoe sizes seem to have an outlier. The outlier will not follow the trend of the rest of the data.

Move to page 1.8.


- Page 1.8 displays the normal probability plot for shoe sizes. Does the plot support your conjecture in question 6? Describe the graph.

Sample Answers: The normal probability plot does support my conjecture. The points appear to be approximately linear, except for the one point on the right. Note that the line does not follow the linear trend but seems to be affected by the outlier.



Teacher Tip: Discussion tying the outlier back to the calculation of the z-scores is very important. If the data were approximately normal, you would expect the very large shoe size to be much smaller.

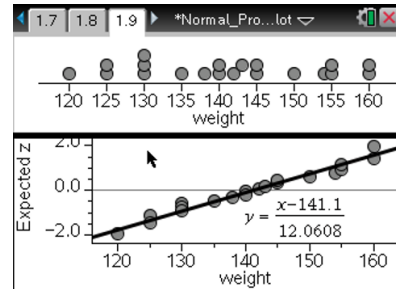


Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☞) getting ready to grab the point. Then press **ctrl**  to grab the point and close the hand (☜). To de-select a point, move the cursor to a white space on the screen, and click. Be sure students recognize that unless they de-select a point they have moved, it will move along with the next point they choose.

Move to page 1.9.

The dot plot at the top of the screen displays the distribution of the weight (in pounds) of a random sample of 20 high school males.

The bottom of the screen displays the normal probability plot generated from the male weights.



8. a. Move the points in the dot plot until the normal probability plot in the lower window is approximately linear. Describe the dot plot.

Sample Answers: The dot plot is approximately symmetric and mound-shaped or approximately normal when normal probability plot is approximately linear.

- b. Move the points in the dot plot until the normal probability plot is concave up. Describe the dot plot.

Sample Answers: The dot plot is skewed left when the normal probability plot is concave up meaning most males' weight have heavier amounts.

- c. Move the points in the dot plot until the normal probability plot is concave down. Describe the dot plot.

Sample Answers: The dot plot is skewed right when the normal probability plot is concave down meaning most males have lower weights.



Teacher Tip: It is possible for distributions of data to be symmetric and mound-shaped but have proportions of data in the tails that differ from the proportions in the normal distribution (68-95-99.7 rule). This would lead to a nonlinear normal probability plot. As an extension to this activity, students can investigate moving the points so the dot plot is approximately symmetric and mound-shaped but the normal probability plot is curved.

9. a. In this activity, you looked at distributions that were approximately symmetric and mound-shaped, skewed left, and approximately symmetric and mound-shaped with an apparent high outlier. If you knew the normal probability plot was approximately linear, what can you conclude about the original data set?

Sample Answers: The distribution of the data is approximately normal.

- b. What would a normal probability plot look like for an approximately normal distribution with a low outlier? Explain your answer.

Sample Answers: The normal probability plot would be approximately linear with a single point that does not follow the rest of the linear pattern in the lower left of the plot.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand that a normal probability plot can help determine whether a distribution of data is approximately normal because:

- Approximately normal data are represented by a linear trend on a normal probability plot.
- Skewed data are represented by a normal probability plot that is either concave up or concave down.
- Outliers appear as points that do not follow the pattern of the rest of the data on a normal probability plot.

Assessment

As an assessment or follow-up, you might want to collect data on these or other variables from your class and explore their distributions and normal probability plots.



TI-Nspire Navigator

Note 1

Name of Feature: Quick Poll

Quick Polls can be given throughout the lesson to check for understanding of the relationship between the appearance of a normal probability plot and the distribution of the data. You can use the results to drive discussion on misunderstandings.

A *Quick Poll* can be given at the conclusion of the lesson. For example, “What would the normal probability plot display for a data set that is skewed left?” or “What should a normal probability plot look like in order to verify that the distribution of a data set is approximately normal?”. You can save the results and show a *Class Analysis* at the start of the next class to discuss possible misunderstandings students might have.