Double Tree

Student Activity

Name ___________________

Class ___________________

Problem 1 – Modeling Tree Growth After 2000

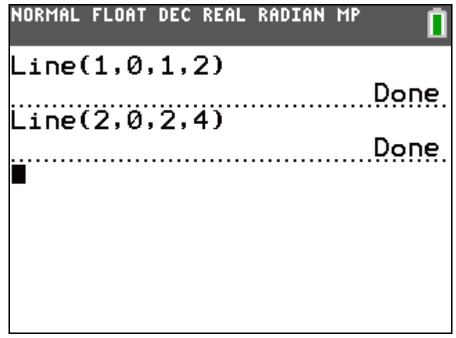
Imagine a tree that doubles its height every year. Sounds like it grows pretty fast! In the year 2001, the tree was 2 feet tall. You can use line segments to model the tree's growth on your calculator. The year 2001 is represented by $x = 1$ and 2002 by $x = 2$.

Draw line segments (**DRAW** menu) to represent the tree's height from 2001 to 2008.

Store the picture as Pic1 (**STO** menu).

The first two commands drawing segments for 2001 and 2002 are shown at the right.

Press GRAPH to view the two trees you drew.



Then enter the command **StorePic Pic1** on the Home screen to save the graph of the segments.

You will notice that all of the lengths are powers of two. Complete the table at the right.

Year	*x* =	Height (feet)	Power of Two
2001	1	2	2^1
2002	2	4	2^2
2003			
2004			
2005			
2006			
2007			
2008			

The heights form a sequence: 2^1, 2^2, 2^3…. You can write this sequence in shorthand using the variable *n*, like this:

$$a_n = 2^n \text{ for } n = 1, 2 \ldots 8.$$

Enter the values of *n* in **L1**. In list **L2**, enter the formula for the sequence, $a_n = 2^n$ but replace *n* with **L1**. Graph the sequence in **Plot1**. Recall Pic1 to view the sequence with the trees you drew.

- How would you describe the overall trend in the graph?

Problem 2 – Modeling Tree Growth Before 2000

- What about the years before 2001? How tall was the tree in 2000? In 1999?

If you read the graph from left to right, the height doubles every year. But if you read the graph from right to left, moving backwards in time, the height is halved each year.

- If $x = 1$ represents the year 2001, what represents 2000? 1999? 1995?

Adjust the window and draw a line segment that is half the 2001 tree's height at $x = 0$.

- Why isn't the segment visible?

Continue drawing line segments to represent the height of the tree in 1999, 1998, 1997, 1996, and 1995, cutting the height of the tree in half each time you step back a year. Save as **Pic2**.

These lengths are also powers of two. Use patterns to complete the table at the right.

Year	*x* =	Height (feet) Decimal form	Height (feet) Fraction form	Power of Two
2000	0	1	1	2^0
1999	–1	0.5	½	2^{-1}
1998				
1997				
1996				
1995				
1994				
1993				
1992				

- What do you notice about the denominators of the fractions in this sequence?

Looking at the table, you will see that these heights also form a sequence: 2^{-1}, 2^{-2}, 2^{-3} ….
We can extend the sequence we graphed earlier to more values of n = –8...8
Use one of the methods described earlier to extend the sequences in **L1** and **L2**.
L1 should list the values of n, and **L2** should list the values of $a_n = 2^n$.
Graph the sequence in **Plot1**. Recall **Pic2** to view the sequence with the trees you drew.

- How does this graph compare to the one in Problem 1?

- Write a rule for evaluating negative powers of 2.

- Test it by evaluating 2^{-12} and 3^{-5} two different ways.

Originally it seemed like a tree that doubled its height every year must grow very fast.

- How much did the tree grow from 1997 to 1998? From 1992 to 1993?

- Does it still seem like the tree grows very fast?

