# Confectionery Delight

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**Abstract:** This activity is an application of differentiation. Students first make physical models using paper and scissors to get a grasp of the problem and to determine how to set it up. They then use the symbolic capacity of their calculator and calculus to determine the dimensions of the box which gives the maximum volume.

**NCTM Principles and Standards:**
**Algebra standards**
a) Understand patterns, relations, and functions
b) generalize patterns using explicitly defined and recursively defined functions;
c) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
d) use symbolic algebra to represent and explain mathematical relationships;
e) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
f) draw reasonable conclusions about a situation being modeled.

**Geometry standards:**
a) Analyze characteristics and properties of two- and three-dimensional geometric shapes and mathematical about geometric relationships
b) draw and construct representations of two- three-dimensional geometric objects using a variety of tools;
c) visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections;

**Problem Solving Standard** build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

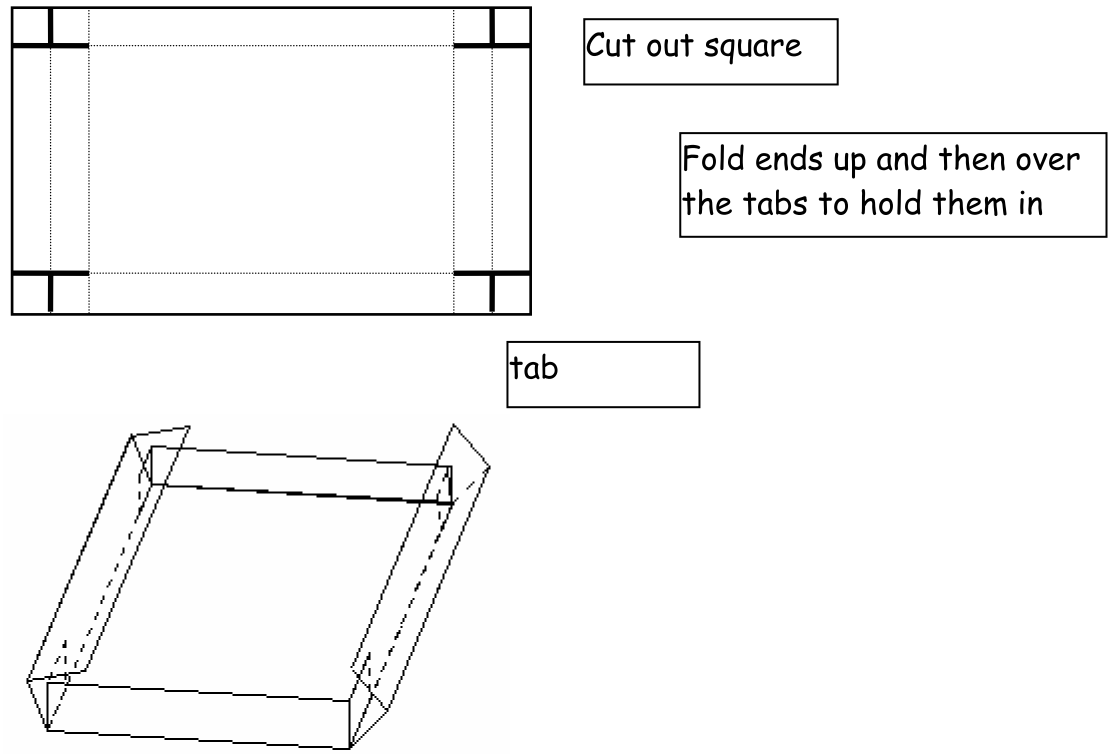
**Reasoning and Proof Standard**
a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;
d) select and use various types of reasoning and methods of proof.

**Representation Standard** : use representations to model and interpret physical, social, and phenomena.

**Key topic:** Applications of Derivative - determining the maximum of a function

**Degree of Difficulty:** Elementary to moderate
**Needed Materials**: Paper and scissors, TI-89 calculators

**Situation:**
The Crispy Cream donut company wishes to make boxes to hold its new confectionery delight. These boxes are to be made from rectangular pieces of cardboard each of which is 8.5 by 11 inches. The box is made by first cutting a small square from each of the corners .

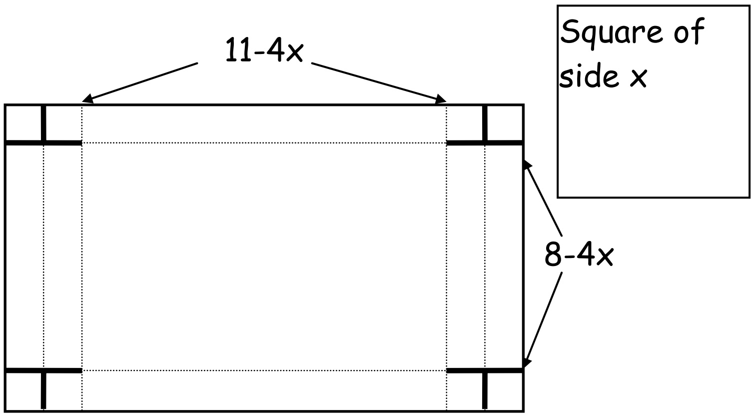



1. Take a 8.5 by 11 inch piece of paper and cut a one inch by one inch square from each of the corners (see the picture). Then cut on the bold lines. Fold on the dotted lines to make a box with the tabs being held down by the sides.
a) What is the height of the box? ( 1 inch)
b) What is the width of the box? (11 – 4= 7 inches)
c) What is the length of the box ( 8.5 - 2 = 6.5 inches)
d) What is the volume of the box? (45.5 square inches)

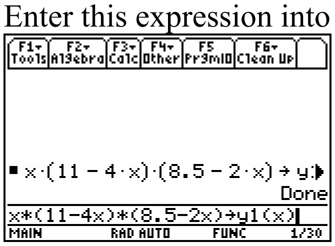
There is another way of making the box – by having the tabs on the other side. Take another piece of the same size paper (rotate it 90 degrees compared to the first one) and also construct a box by cutting out one inch squares from each corner. ). Then cut on the bold lines. Fold on the dotted lines to make a box with the tabs being held down by the side.
a) What is the height of the box? ( 1 inch)
b) What is the width of the box? (8.5 – 4= 4.5 inches)
c) What is the length of the box ( 11 -2 = 9 inches)
d) What is the volume of the box? (40.5 square inches)

Obviously it makes a difference which way the piece of paper if oriented. It undoubtedly also makes a difference what size square is cut out.  We can use calculus to compute the size of the square which gives us the maximum volume.

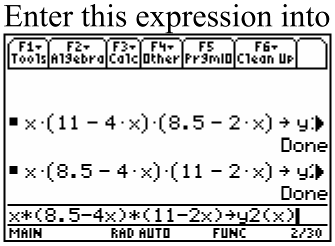
Like before we'll consider the two possible orientations of the paper and determine which size square gives us the largest volume.



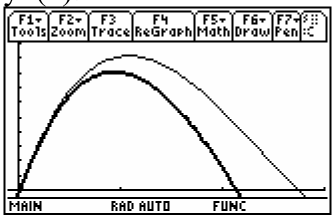

Let the edge of the square be x. Then, following the above diagram,
a) What is the height of the box? ( x inches)
b) What is the width of the box? (11 – 4x inches)
c) What is the length of the box ( 8.5 –2x inches)
d) What is the volume of the box? ( x(11-4x)(8.5-2x) square inches)
Enter this expression into your calculator and store it in y1(x)



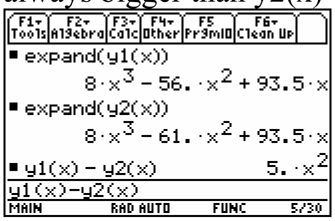

Now, change the orientation of the paper so that the tabs are made from the other sides. Now…
a) What is the height of the box? ( x inches)
b) What is the width of the box? (8.5 – 4x inches)
c) What is the length of the box (11 –2x inches)
d) What is the volume of the box? (x(8.5-4x)(11-2x) square inches)
Enter this expression into your calculator and store it in y2(x)



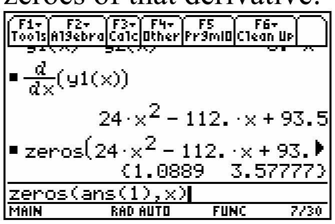
Compare two graphs of the functions and it appears that y1(x) is greater than or equal to y2(x).



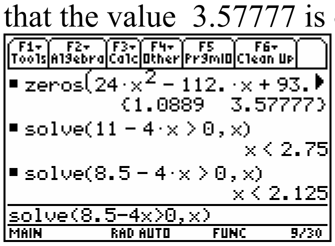

By expanding the expressions for the two volume functions, one can tell that y1(x) is always bigger than y2(x) – because they differ by the term 5x^2.



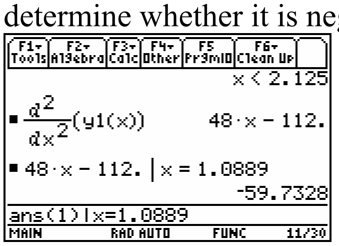

We now can use the calculator to find the value of x which gives us the maximum volume for our function y1(x). We first find the first derivative of y1(x) and then find the zeroes of that derivative:




Since we know that each of the sides of the box must have a positive length, we can see that the value 3.57777 is out of our domain for this problem:



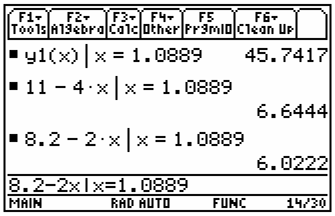

One way to determine whether we have a maximum is to compute the 2nd derivative and determine whether it is negative at the zero of the first derivative:




One should always check the endpoints of the function to insure that we have identified the value which gives us the absolute maximum. Here the endpoints are x = 0 and x = 2.125 and both of those give zero volumes as the graph indicated.

Finally, what is the maximum volume? To calculate that we just need to evaluate our volume function, y1(x) at the zero of the derivative:



Our box has a volume of 45.7417 square inches and its dimensions are 1.0889 X 6.0222 X 6.6444.
The maximum volume's value is quite close to the one we got when we physically cut out the square from the paper!