EXPLORATIONS™

Activity 6

Exploring the Exponential Function

Objectives

- Differentiate between exponential growth or decay from an equation
- Identify the coefficient in an equation that represents the rate of growth/decay
- Explain the effect of changes in the values of *A*
- Compare two graphs by looking at their equations
- Find an approximate model of a data set involving exponential growth

Introduction

When you deposit money in a savings account, it grows by a constant percentage. Suppose one bank has an interest rate of 6% compounded annually. If you deposited money in this bank, each year the bank would add to your deposit the interest earned—6% of the money in your account. If you kept that money in the bank for a number of years, your account would show ***exponential growth***.

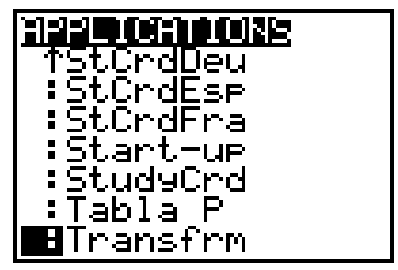
On the other hand, if you drop a ball and let it bounce without touching it, as you did in Activity **4**, the ball rebounds less and less with each bounce. Each bounce rebounds at a percentage of the previous height. This is an example of ***exponential decay***.

If a cup of hot chocolate was allowed to cool, the temperature of the hot chocolate over the cooling period would show exponential decay.

When the value of a variable increases or decreases by a constant *percent*, the variable is said to change exponentially. In these activities, you will explore exponential functions of the form $y = ab^x$. In this form, b is called the growth factor. You will also identify graphs of exponential growth or decay from equations and will approximate models of exponential growth.

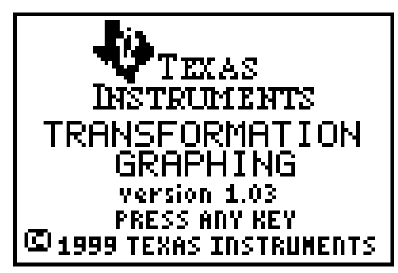
Investigating the Effect of *A* and *B* on the Graph of $Y = AB^X$

1. Press [APPS] and select **Transfrm**.



2. Press any key (except [2nd] or [ALPHA]) to start the Transformation Graphing App.

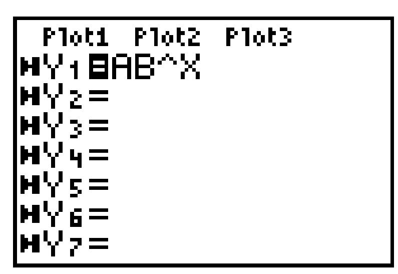
   Note: *If you do not see the screen shown, select* **Continue**.



3. In **Func** mode, press [Y=] to display the **Y=** editor. Clear any functions that are listed, and turn off any plots.

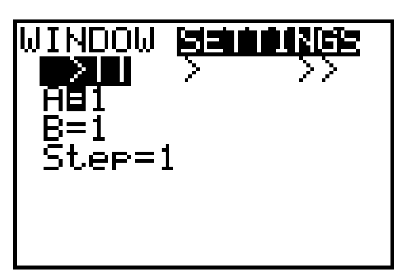
4. At **Y1=**, enter **AB^X**. Press [ALPHA] **A** [ALPHA] **B** [^] [X,T,Θ,*n*].

   If Play-Pause mode (**>||**) is not selected at the left of **Y1**, press [◄] until the cursor is over the symbol; then press [ENTER] until the correct symbol is selected.

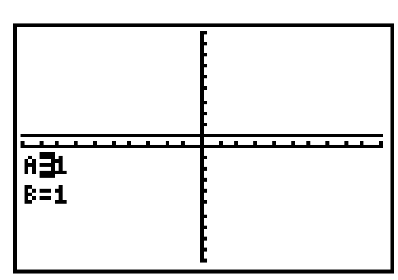


5. Press [WINDOW] [▲] to display the **SETTINGS**.

   Enter the values shown. To make these selections, press [▼] **1** [▼] **1** [▼] **1**. These settings define the starting values for the coefficients and the increment by which you want the coefficients to change.



6. Press [ZOOM] **6** to select **6:ZStandard** and display the graph.



Questions for Discussion

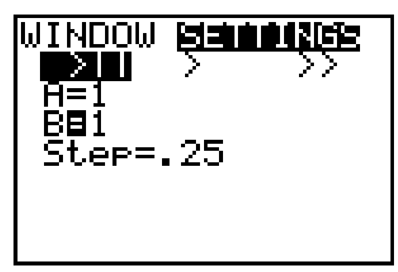
1. The graph appears to be a line. Why? Explain your answer.

2. If *B* remains 1 and *A* changes, what do you think will happen to the graph? Make a hypothesis, and then press the cursor keys (either [◄] or [►]) a number of times. How did the graph change?

Investigating *B* > 1

1. Press [WINDOW] and select **SETTINGS**.

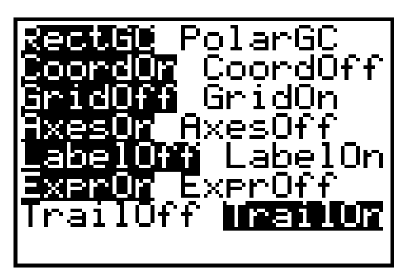
   Enter the values shown. These settings define the starting values for the coefficients and the increment by which you want the coefficients to change.



2. Press [2nd] [FORMAT] to access the **FORMAT** settings. Highlight **TrailOn** and press [ENTER]. With **TrailOn** active, as the value of *B* is changed and the graph redrawn, a dotted line remains in the location of the last curve.

   Note: **TrailOn** *is a feature that exists only with the Transformation Graphing App.*

   To clear the trails, press [2nd] [DRAW] **1** to select **1:ClrDraw** or repeat the steps above and highlight **TrailOff**.



3. Press [GRAPH] to return to the graph screen.

4. Highlight the **B=**. Press [▶] a few times to increase the value of *B*. Pause after each increase to notice the change in the graph. You only need to change the value about 5 times.

   What happens to the graph as *B* increases?

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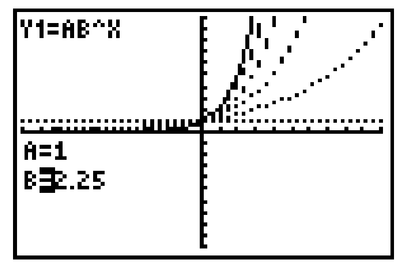
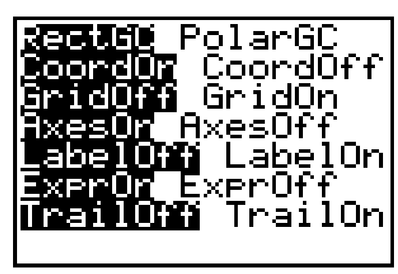


The value of *B* controls the rate at which the function grows and is called the growth factor. As *B* increases, the function bends upward more steeply.

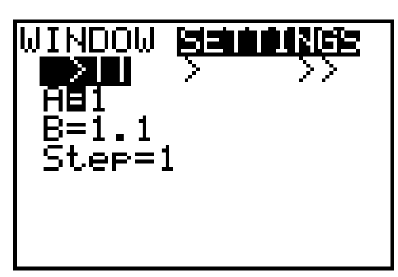
So far you have only investigated values of $B > 1$. We will save our investigation of $B < 1$ for later in these activities.

Investigating the Effect of *A*

Before starting this investigation, press [2nd] [FORMAT] to set the **FORMAT** settings as shown.

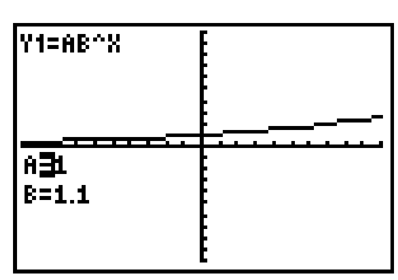


1. Press [WINDOW] [▲] to display the **SETTINGS** screen. Enter the values shown.



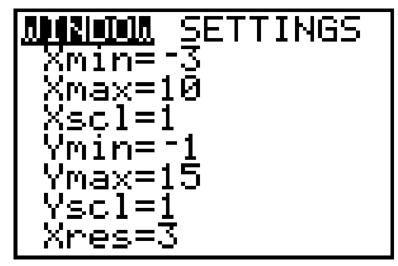
2. Press [GRAPH] to return to the graph screen. Highlight **A=**.

3. Press [▶] a few times to see the effect on the graph as the value of *A* increases. Pause each time the curve is re-graphed and take note of the change. Pay special attention to the value of the *y*-intercept with each move.



4. Press [WINDOW] and enter the **WINDOW** settings shown.

5. Press [GRAPH] to redisplay the graph with the new settings.



6. Enter the values **2**, **6**, and **10** for *A*.

   What effect does *A* have on the graph of $Y = AB^x$?

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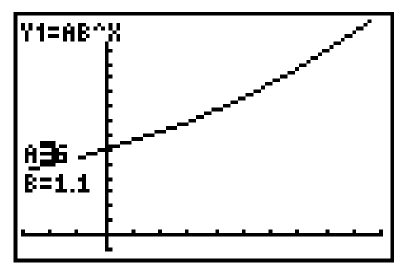


The *y*-intercept of an exponential function is frequently called the starting value of the dependent variable, the variable that is growing exponentially. For example, if you deposited \$100 in a bank, at time $t = 0$ (when you first deposited the money), you would have \$100 ($y = 100 \bullet b^0 = 100 \bullet 1$) in your account.

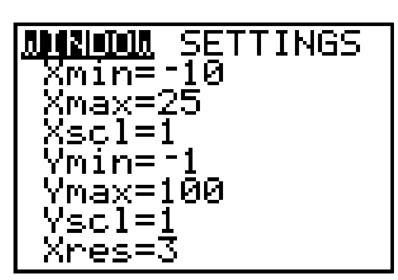
To set up an exponential model for the growth of your money, you would let $A = 100$, the starting value. The value of *B* would reflect the interest rate.

Revisiting *B*

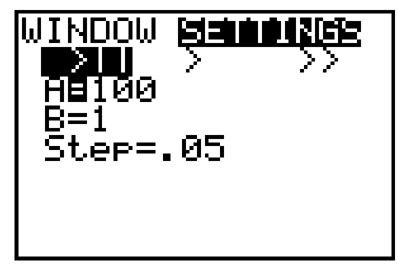
Now look at the cup of hot chocolate as a second example. When it was made, the hot chocolate was 60ºC, too hot to drink. So it was placed on a table outside where the temperature was 0ºC. As it sat outside, the hot chocolate cooled at an exponential rate, called *exponential decay*. Expressed as $Y = AB^x$, the value of *A* would be 60, the starting temperature. *B* reflects the rate at which the hot chocolate was cooling, and *x* would be the length of time the hot chocolate was cooling.

You have considered the model for exponential decay but have not viewed its graph. The next investigation will show you how an exponential decay curve looks.

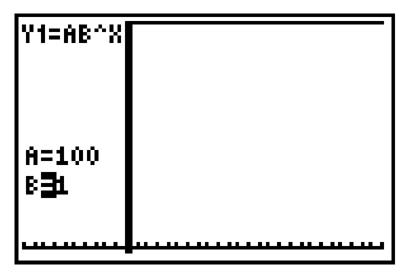
1. Press [WINDOW] and enter the values shown.



2. Highlight the **SETTINGS Menu**. Enter the values shown.

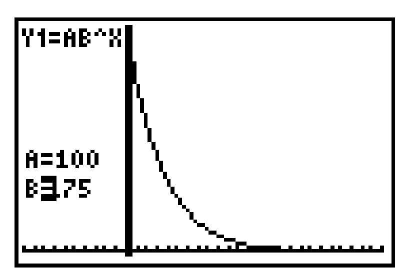


3. Press [GRAPH] to return to the graph window.



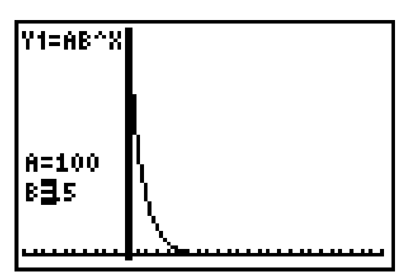
4. Press [▼] until the **B=** is highlighted.

5. Press [◄] several times to lower the value of *B*. Observe values of *B* between 1 and 0. Pause after each graph change and notice the effect on the graph.



Note: *You might want to enable* **TrailOn**. *If you do, be sure to turn that feature off when you finish this particular part of the activity.*

What effect does lowering the value of *B*, in the range $0 < B < 1$, have on the graph of the exponential function $Y = AB^X$?



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Note: B *cannot be negative. Allow* B *to be negative and investigate the function values with the table feature of your graphing handheld.*

Assuming $A > 0$ and $0 < A < 1$, explain why the value of *Y* can never be negative for $Y = AB^X$.

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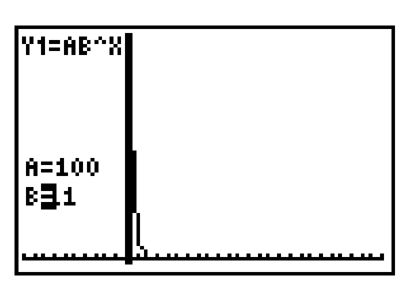
Curves that approach a value, but never reach it are said to be ***asymptotic***. The exponential equation, $Y = AB^X$ is asymptotic to zero because it approaches zero as *x* gets larger but never reaches zero.



How Are You Doing?

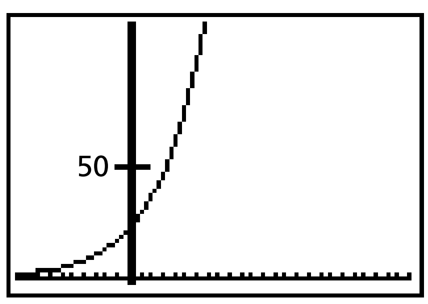
Each graph is an example of an exponential function. For each, tell:

a. whether the graph is an example of growth or decay.

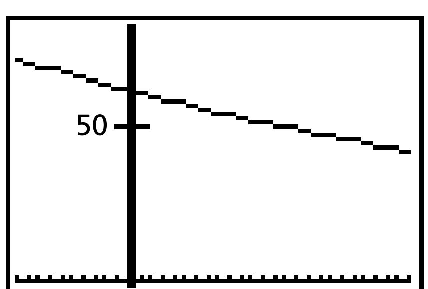
b. what the approximate value of *A* might be.

c. whether $B < 1$ or $B > 1$.

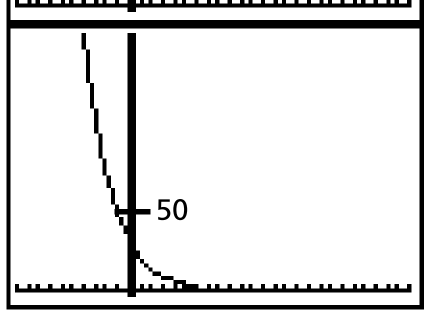
1.



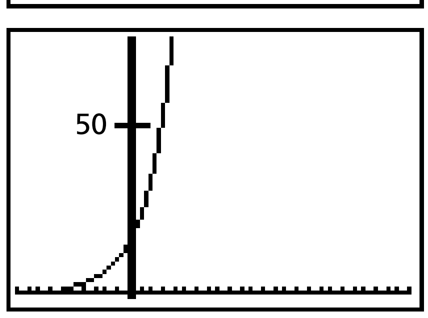
2.



3.



4.



5. Which graph above, **2** or **3**, has the larger value for *B*? Explain your answer.

6. Which graph above, **1** or **4**, has the larger value for *B*? Explain your answer.

Student Worksheet

Name ______________________

Date ______________________

The table shows the population of the world, in billions, since 1940. Use the Transformation Graphing App to develop an exponential model for the data.

Year	Population (in billions)
1940	2.30
1950	2.52
1960	3.02
1970	3.70
1980	4.44
1990	5.27
2000	6.01

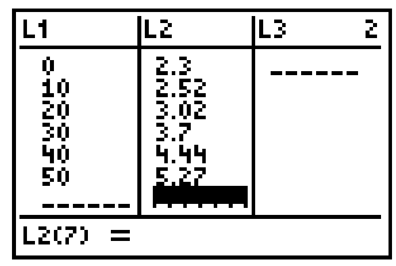
Source: Census 2000, data at http://www.census.gov/ipc/www/world.html.

Before you enter the data, think about the parameters of the exponential function $Y = AB^X$. The value A is the starting value of the dependent variable, in this case, world population in billions. The starting value is 2.3 billion for the first year listed (1940). The starting value of your independent variable should be zero, so set 1940 equal to 0. Then, subtract 1940 from each date in the table.

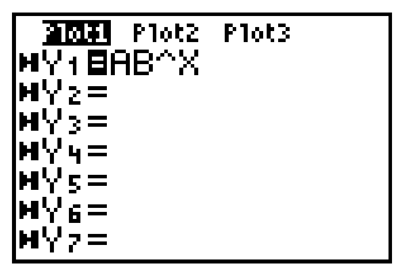
Year	Translated Year (x)	Population (in billions) (y)
1940	0	2.30
1950	10	2.52
1960	20	3.02
1970	30	3.70
1980	40	4.44
1990	50	5.27
2000	60	6.01

1. To enter the data in the stat list editor, press [STAT] **1** to select **1:Edit**. Enter the values from the table into two empty lists. Clear two lists if no lists are empty.

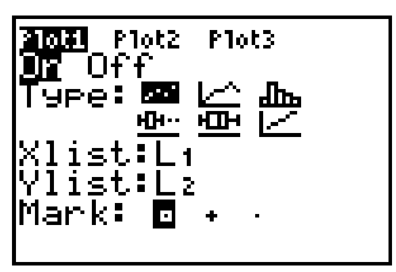
   Note: *To clear a list, move up to highlight the list name, and then press* [CLEAR] [ENTER].



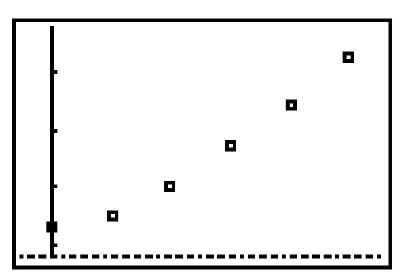
2. Press [Y=] and clear all functions. Enter the general form of the exponential function.



3. To display the plot, press [2nd] [STAT PLOT] **1** to select **1:Plot1**. Turn the plot on, and select the settings as shown. Press [ZOOM] **9** to select **9:ZoomStat** and display the plot.

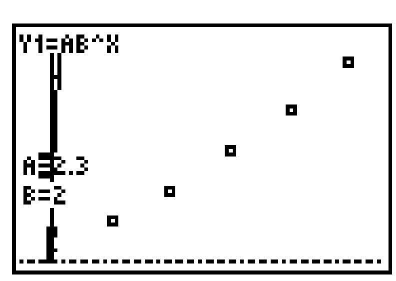


From the plot, it appears that an exponential model might be appropriate.



4. Make sure that the Transformation Graphing App is installed and in Play-Pause Mode. Press [GRAPH] to redraw the plot with the graph.

5. Select starting values for *A* and *B*. As an example let **A=2.3** and **B=2**. This gives you a starting point in your investigation.



Should the value of *B* be larger or smaller than that shown in the graph? Explain.

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6. Revise your values for *A* and *B* until you have a model that approximates the plot of the data. (Keep in mind what parameter *A* represents. What should be the value of *A*? __________) Express *B* to the thousandths place to get a good model.

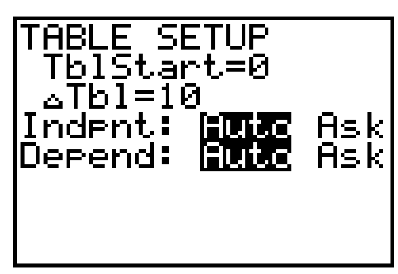
What equation did you decide is a reasonable model for the data?

Y = ___________

7. When you have a reasonable model, use the table feature of your graphing handheld to estimate the world population in the year 2010.

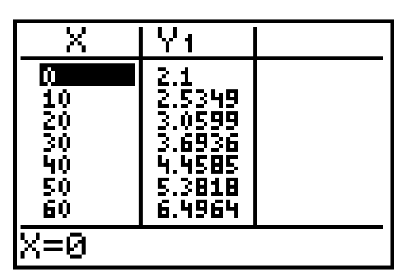
Hint: *If 1940 was entered as* **0** *in our table, how should you enter the year 2010 in the table?*

To use the table feature, press [2nd] [TBLSET] and enter the settings shown.



To read the table, press [2nd] [TABLE].

Note: *Your table might have different values, depending on your model.*



8. What does your model predict the population will be in 2010?

____

9. What does you model predict the population was in 1930?

____

Modeling Practice

The population of the mystical city of Transform is shown in the table.

Note: *Remember to start your independent variable at 0.*

Year	Population
1970	125,000
1975	201,300
1980	324,000
1985	522,000
1990	841,000
1995	1,354,000
2000	2,214,000

10. Find a model for the population of Transform.

____

11. What will the population be in 2005?

____

12. At what rate is the population growing?

____

Teacher Notes

Activity 6

Exploring the Exponential Function

Objectives

- Differentiate between exponential growth or decay from an equation
- Identify the coefficient in an equation that represents the rate of growth/decay
- Explain the effect of changes in the values of *A*
- Compare two graphs by looking at their equations
- Find an approximate model of a data set involving exponential growth

Materials

- TI-84 Plus/TI-83 Plus

Time

- 90 minutes

The purpose of this activity is to start a study of the exponential function. Students will be able to explain that the value of *B* affects the rate at which the function grows or decays. The magnitude of *B* is not the focus of this investigation; however, students should realize that the value of *B* affects the rate at which the function grows.

This activity is appropriate for students in Algebra 2 although it could be done with students in Algebra 1.

Optional Presentation Method

Activities **6** and **7** could be divided into three sections: growth, decay, and non-zero asymptotes. If you decide to use this division, the sections would be as follows:

- *Growth*: Activity **6** from start to the section Revisiting *B*.
- *Decay*: Start from the section Revisiting *B* and continue into Activity **7**. End in Activity **7** with the section Modeling the Experiment: Casting Out Sixes with Special Number Cubes.
- *Non-zero Asymptotes*: This section would start where the previous section ended and continue for the rest of the activity. Any "cooling" experiment could be used for this homework.

In this activity, students are presented with the idea of the exponential decay through the cooling of hot chocolate, but they are not presented with the idea of asymptotes. In the hot chocolate problem, the starting temperature of the beverage is 60°C and it cools to 0°C. In this way the asymptote is at $y = 0$. Investigating decay completely with non-zero asymptotes would be too much for one set of activities, especially for students in Algebra 1 courses, where the topic of non-zero asymptotes is generally not covered. Non-zero asymptotes are presented in Activity **7**.

Answers

Investigating the Effect of A *and* B *on the Graph of* Y = AB^x: *Questions for Discussion*

1. When $A = B = 1$, you have the equation $y = 1(1)^x$. No matter what value x takes, 1^x will remain 1. Thus the equation becomes $y = 1(1)$ or the line $y = 1$. In this case there is no growth or decay.

2. Because $B = 1$, you will continue to have the line $Y = A$. The general form of the equation becomes $Y = AB^x = A \times 1 = A$.

   B is the growth factor; as long it remains 1, there is no change.

Investigating B > 1

4. As B increases, the curve gets steeper and the function grows at a faster rate.

Investigating the Effect of A.

6. The parameter A is the y-intercept of the curve. The y-intercept is the value when $x = 0$. ($Y = AB^0 = A \times 1 = A$).

Revisiting B

5. When $0 < B < 1$, the values y decrease as x grows. The closer B is to 0, the faster the y-values decrease and thus the faster the rate of decay of the graph.

   Because $0 < B < 1$, the more you multiply B by itself, the smaller the term becomes, but it can never be less than 0. As an example use 0.9^1, 0.9^2, 0.9^3, …, 0.9^{10}, when you evaluate them you get 0.9, 0.81, 0.729, …, 0.349. The value of each successive term is getting smaller but will never reach 0. Thus, as long as A is not negative, there is no way for AB^x to ever be negative, only very small.

How Are You Doing?

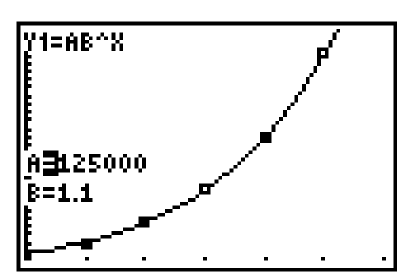
1. **a.** Growth **b.** 20 **c.** $B > 1$
2. **a.** Decay **b.** 60 **c.** $B < 1$
3. **a.** Decay **b.** 25 **c.** $B < 1$
4. **a.** Growth **b.** 15 **c.** $B > 1$
5. The value of *B* is greater in 2, because the curve doesn't bend as much, it decays more slowly.
6. The value of *B* is greater in 4, because the curve bends faster, it grows faster.

Student Worksheet

5. The value for *B* should be smaller. The graph is growing too quickly.
6. Answers will vary. $y = 2.2(1.019)^X$ could be a reasonable model. Answers in the neighborhood of $y = 2.2(1.0174)^X$ are exceptionally good. With exponential growth, the number of decimal places is very important.
8. Approximately 7.36 billion; Make sure that students label the answer as billions.
9. Approximately 1.85 billion; Make sure that students label the answer as billions.

Modeling Practice

10. A reasonable model is $y = 125000(1.05)^X$ assuming 1970 becomes 0.



11. The population in 2005 will be approximately 3,524,000.
12. The population is growing at approximately 10% per year.