

Activity 10

Dilations in the Plane

Objective

- To use the **Dilation** tool on the Cabri® Jr. application
- To investigate the properties of a dilation
- To extend the concepts of dilations to the coordinate plane

Cabri® Jr. Tools



Introduction

Dilations constitute a fourth type of transformation studied in geometry. In this activity you will learn to use the Dilation tool on the Cabri Jr. application. In addition, you will investigate the properties of dilations and the effects of dilations in the coordinate plane.

This activity makes use of the following definitions:

Dilation — a transformation that expands or shrinks a figure by a given factor

Pre-image — the original object that is to be transformed



Image — the new object created by applying the conditions of a transformation



Part I: Properties of a Dilation


Construction


I. Create a dilation factor tool.


In order to use the Dilation tool, you will need to identify a dilation factor, a number that determines the size of the dilation. While any numeric value or measurement can be used for this, you will create a Dilation Factor tool that will allow you to change the value of the dilation factor dynamically.


  Draw \overline{DG} near the bottom of the screen, with point D near the left edge of the screen and point G near the right edge.


  Construct the midpoint, E , of \overline{DG} .

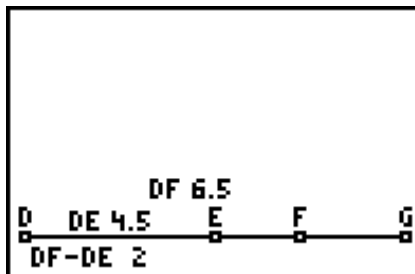
 **A** Construct a point F on \overline{DG} between points E and G .

 **A** Measure and label the distances DE and DF .


 **A** Use these measures to calculate $DF - DE$. Label this value.


 Move point F to verify that it will cause the value of $DF - DE$ to display both positive and negative values.


 Move point F until the value of $DF - DE$ is approximately 2.

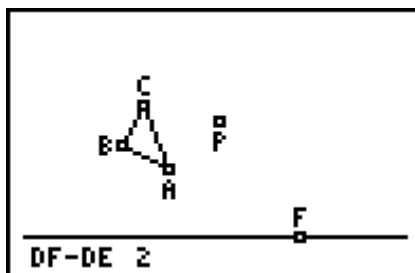


II. Draw a triangle to dilate and a point that will be the center of the dilation.

 Hide points D , E , and G , and the measurements of \overline{DE} and \overline{DF} .

 **A** Draw a small scalene $\triangle ABC$ on the left side of the screen.


 **A** Draw point P near the center of the screen to use as the center of the dilation.

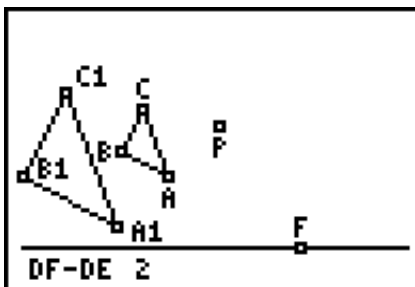


III. Dilate the triangle.

To use the **Dilation** tool, perform the following steps.







- Select the item to be dilated.
- Select the center of dilation.
- Select a numerical value to be the dilation factor.

 Dilate $\triangle ABC$ using point P as the center of dilation and the calculated value $DF - DE$ as the dilation factor. Label the corresponding vertices of $\triangle ABC$, A_1 , B_1 , C_1 .



Note: If part or all of the image does not appear on the screen, adjust the location of point P and/or $\triangle ABC$ until the image appears on the screen.

Exploration

-   Observe the relationship between the pre-image and the image formed by the dilation (with a dilation factor of 2) by dragging the vertices and sides of $\triangle ABC$, $\triangle ABC$ itself, and point P . Use various measurement tools (**Distance and Length**, **Angle**, and **Slope**) to explore any relationships you think exist.
-   Repeat the above exploration using different (fixed) dilation factors. Be sure to investigate dilation factors that are greater than 1, equal to 1, between 0 and 1, and negative.
-   Construct the lines of perspective connecting corresponding points using the outside most vertices. (For this example $\overleftrightarrow{AA'}$ and $\overleftrightarrow{CC'}$.) Repeat the above exploration while point F is animated.

Questions and Conjectures

1. In the creation of the dilation factor tool, the value $DF - DE$ was calculated. Since this is equal to FE , why do you think you did not simply use the distance FE as the dilation factor?
2. Make a list of the properties of the pre-image that are preserved in the image when the dilation factor is 2.
3. Which of the properties that you listed in Question 2 are true for all dilations? Which properties depend on the value of the dilation factor? Be sure to note any differences when the dilation factor is greater than 1, equal to 1, between 0 and 1, and negative.
4. Make a conjecture about how the **Dilation** tool determines the location of the image. Explain how you would test your conjecture.

Extension




How are transformations formed using the **Rotation** tool related to transformations done using the **Dilation** tool? How are transformations formed using the **Symmetry** tool related to transformations done using the **Dilation** tool? Explain your reasoning.

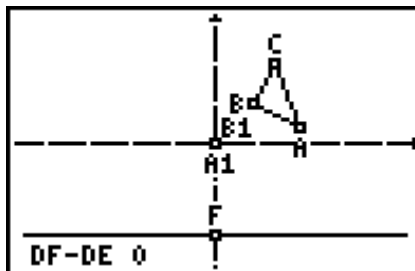
Part II: Dilations in the Coordinate Plane

Construction

Adjust the construction to investigate dilations in the coordinate plane.



Continue using the previous construction.

-  Set the dilation factor to zero.
-  Show the axes on the screen and drag the origin to the center of the screen.
-  Move the following:
 - Point P to the origin.
 - $\triangle ABC$ to the first quadrant.
 - The dilation factor segment to the bottom of the screen.
 - The measurement and label of $DF-DE$ to the bottom of the screen.



Note: Perspective lines are not shown.

Exploration

-   Observe the relationship between the coordinates of the vertices of the pre-image and those of the image formed by the dilation centered at the origin. Be sure to investigate both positive and negative dilation factors.

Questions and Conjectures

Make a conjecture about the relationship between the coordinates of the vertices in the pre-image triangle and the image triangle formed using a dilation through the origin. Is your conjecture true for any dilation factor? Explain your reasoning.

Teacher Notes



Activity 10

Dilations in the Plane

Objective

- To use the **Dilation** tool on the Cabri® Jr. application
- To investigate the properties of a dilation
- To extend the concepts of dilations to the coordinate plane

Cabri® Jr. Tools



Additional Information

A great way to summarize this activity would be to develop a working definition for dilation as a whole class activity.

In Part II of this activity, it is possible to construct the dilation factor tool on the x -axis.

Part I: Properties of a Dilation

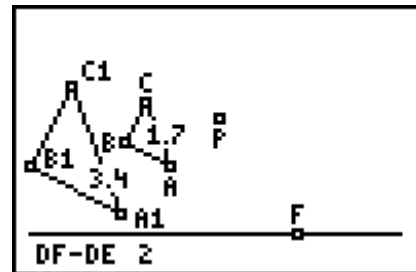
Answers to Questions and Conjectures

1. In the creation of the dilation factor tool, the value $DF - DE$ was calculated. Since this is equal to FE , why do you think you did not simply use the distance FE as the dilation factor?

FE is the distance between the points F and E . This value is always positive. In order to have dynamically occurring values for the dilation factor that include negative numbers, a calculation must be used.

2. Make a list of the properties of the pre-image that are preserved in the image when the dilation factor is 2.

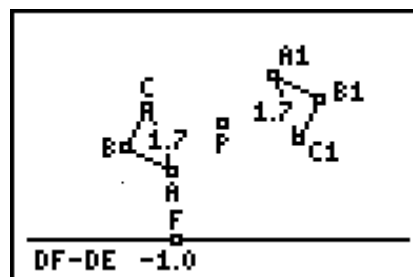
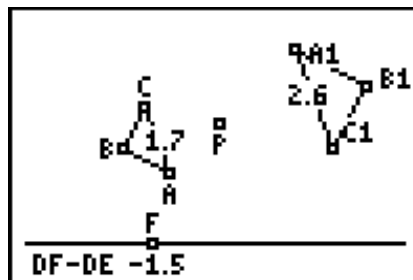
Students should discover that the image is not congruent to the pre-image but is similar to it. This means that corresponding angles are equal, the corresponding sides have the same ratio as the dilation factor, and the areas of the respective images have a ratio of the square of the dilation factor. Students might also discover that the slopes of the sides of the image and pre-image are the same.



3. Which of the properties that you listed in Question 2 are true for all dilations? Which properties depend on the value of the dilation factor? Be sure to note any differences when the dilation factor is greater than 1, equal to 1, between 0 and 1, and negative.

All of the above are true with the following exception. Distances from the pre-image and the image have the same ratio as the dilation factor for positive factors. If the dilation factor is negative, then the figure is rotated 180° as well as dilated. The ratio becomes the absolute value of the dilation factor because distance measurements are always positive.

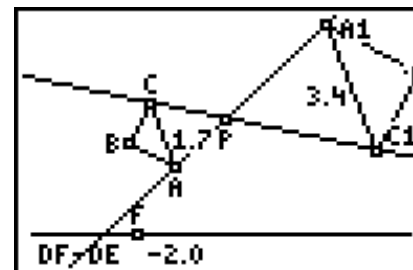
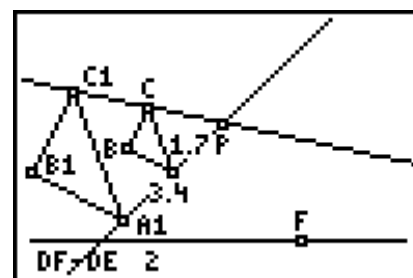
When the dilation factor is 1 or -1, the image triangle is congruent to the pre-image triangle.



4. Make a conjecture about how the **Dilation** tool determines the location of the image. Explain how you would test your conjecture.

The location of the image point will be on the line containing the pre-image point, the lines of perspective, and the center. The distance from the image point to the center is equal to the distance from the pre-image point to the center multiplied by the absolute value of the dilation factor. If the dilation factor is positive, the image point and its pre-image point are on the same side of the center.

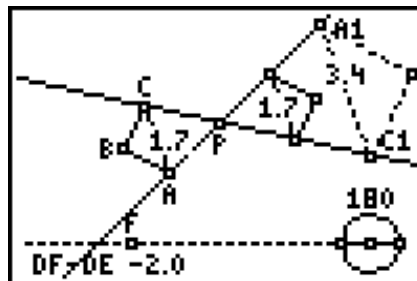
If the dilation factor is negative, the image point and its pre-image point are on opposite sides of the center.



Answers to Extension

How are transformations formed using the **Rotation** tool related to transformations done using the **Dilation** tool? How are transformations formed using the **Symmetry** tool related to transformations done using the **Dilation** tool? Explain your reasoning.

Dilations preserve orientation whether the dilation factor is positive or negative. The figure shows a dilation by a factor of -1 through point P and a rotation of the pre-image around point P . Rotating the pre-image around the center of dilation for a -1 dilation demonstrates that the same transformation can be accomplished using a rotation of 180° . This motion is sometimes called a *half-turn*.

**Part II: Dilations in the Coordinate Plane****Answers to Questions and Conjectures**

Make a conjecture about the relationship between the coordinates of the vertices in the pre-image triangle and those of the image triangle formed using a dilation through the origin. Is your conjecture true for any dilation factor? Explain your reasoning.

The figures below show numerically what happens to the coordinates of a vertex of the triangle using two different dilation factors: 2 and -1 . In general, a dilation by a factor of n through the origin produces the mapping $(x, y) \rightarrow (nx, ny)$.

