EXPLORATIONS

Activity 10

Coffee Break

Economists often use math to analyze growth trends for a company. Based on past performance, a mathematical equation or formula can sometimes be developed to help make predictions about future results. As you will see, it is relatively easy to build a mathematical equation that fits a given set of data. The trick is finding a mathematical model that has predictive value—one that can help us plan for events in the real world.

Introduction

In this experiment, you will use the TI InterActive!™ browser to access data relating to the openings of new Starbucks Coffee Company shops in the last decade. What mathematical model best describes the growth of Starbucks shops with time? To what extent can a modeling equation be used to predict how many shops there will be in the future?

Equipment Required

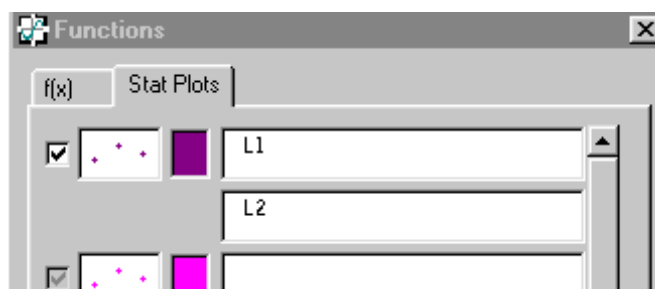♦ A working Internet connection

♦ TI InterActive! software

Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.

2. Title your new document *Coffee Break* and add your name and the date. Click the

    Save button ▢ to save and name your document.

3. Click the Web Browser 🌐 to open the TI InterActive! browser. Click on the Data

    Sites button 📇. Under the **Activity Book Links** category, click on
    TI InterActive! Data Collection and Analysis. Choose **Activity 10: Coffee Break**.

4. Once the page has been loaded in the browser, scroll down to the heading *The Company* and click *News*. Then click *Company Overview*. Finally, click *Timeline*.

5. Scroll through the Starbucks timeline narrative and note the total number of coffee shop locations each year, starting with 1987 and ending with 1997. Record this data in the table provided.

Year	Years since 1986	Total Number of Starbucks Locations
1986	0	1
1987	1	17
1988	2	
1989	3	
1990	4	
1991	5	
1992	6	
1993	7	
1994	8	
1995	9	
1996	10	
1997	11	

Recording the Data

1. Click the List button , and then click the empty cell at the top of list **L1**. Type the starting time value, **0**, and then press the down arrow key to move to the next cell. Continue entering the number of years since 1986 until you have entered all of the time values into **L1**.

2. Click the empty cell at the top of list **L2**. Type the total number of Starbucks locations for each corresponding time value in **L1**.

3. Click the Scatter Plot button then click the Stat Plots tab. In the uppermost text box, type **L1** to specify it as the list containing the *x* coordinates. Press the Tab key to move to the second text box, and type **L2** to specify the list containing the *y*-coordinates.

4. Press Enter, and then click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.

   The plot of number of coffee shops versus the number of years since 1986 should be curved upward.

If you are not satisfied with your results, check your data against the Web site again.

If you are satisfied with your data, you can make a sketch of the coffee shops versus the number of years since 1986 data that you collected on a blank grid in the Appendix. Label the horizontal and vertical axes on your sketch.

5. Click the Save to Document button to save the graph in your TI InterActive!™ document.

Analysis and Questions

1. When the relationship can be expressed in the form $y = A B^x$, with $B > 1$, y is said to *grow exponentially* with x. The constant B is called the *base*. Notice that when $x = 0$, the y value is equal to A. Use your data table to determine the value of A and record it below:

   $A =$

2. In order to find an exponential model for data you collected, you will need to find an appropriate value for B using the guess-and-check method. Click the f(x) tab in the upper left corner of the Functions dialog box. For exponential growth, the model demands that $B > 1$, so start with an initial guess of $B = 2$. Type **f(x):= A*2^x** in the uppermost text box of the f(x) tab, using the numerical value of A from above. Press Enter to superimpose the graph on the plotted data.

   It is unlikely that your first guess for the value of B produced a model that matched the data closely. Click in the text box of the f(x) tab again and edit the exponential equation, replacing the old value, $B = 2$, with your new guess for B. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a B-value that models the data well and record it in the space below (give your answer to two decimal places):

   $B =$

3. Can the equation you just found be used to predict how many new Starbuck locations opened after 1997? Click **Table** to display the table screen. Click the Table Setup button . Set the Independent Mode to **Ask** and the Dependent Mode to **Auto**, as shown on the next page, then click **OK**.
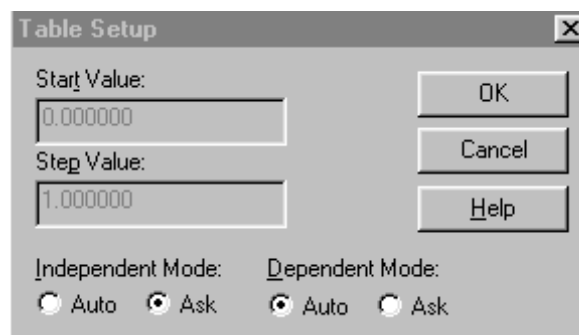
An empty table is displayed. As you enter values for the independent variable, *x*, values for the dependent variable, *y*, will be generated using the equation you entered in the Function dialog box. To start, type the number **12** (corresponding to 1998) into the first *x*-cell, then click on the corresponding *y*-cell to display the number of coffee stores your model predicts for that year. Repeat for *x = 13* (corresponding to 1999), and record your values in the second column of the table below.

Year	Predicted Number of Locations	Actual Number of Locations
1998		
1999		

How do your predictions compare with the actual number of Starbucks locations that existed in 1998 and 1999? Follow steps **3** and **4** in the *Collecting the Data* section of this activity to return to the Starbucks web site and find out how many locations there were for these two years. Record this information in third column of the table above.

4. Why do you suppose there was such a big difference between your predicted values and the actual number of coffee shop locations? What real-world features of Starbucks growth were not accounted for in the exponential modeling equation?

5. The advanced statistical features of TI InterActive!™ allow you to create a modeling equation that more completely describes the Starbucks growth phenomenon you have been considering. The modeling equation is called a *logistics curve* and takes into account the fact that growth of this sort eventually levels out or reaches a *carrying capacity*. Because the logistics growth formula is very complicated, you will rely on the regression features of TI InterActive! to find the modeling equation for us.

   a. Click the Function Table close box ☒ (click **No** when asked if you want to insert a copy of the table into the document) and the Graph close box ☒ to return to the Data Editor.

   b. Click Statistical Regressions ▦.

   c. Click the down arrow ▾ next to Calculation Type, scroll down the list and click on **Logistic Regression**.

   d. In the text box labeled **X List**, type **L1**; in the box labeled **Y List**, type **L2**.

   e. Click **Calculate** to find the regression equation and its variables. Click **Save Results**. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

6. Close the Data Editor. Make sure the graph is positioned below the equation in the TI InterActive! document, then double-click on the graph to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regeq** and then press Enter. TI InterActive! graphs the equation that was created as the Statistical Regressions Result together with your original guess-and-check exponential model. Which equation seems to fit the data better?

7. How well does the logistic model predict how many new Starbucks locations opened after 1997? Click 〔 Table 〕 again, then click the Table Setup button 〔▦〕. Set the Independent Mode to **Ask** and the Dependent Mode to **Auto**, as before, then click **OK**. Type the number **12** (corresponding to 1998) into the first *x*-cell, then click on the corresponding *y*-cell to display the number of coffee stores your model predicts for that year. Repeat for *x* = 13 (corresponding to 1999), and record your values in the second column of the table below.

Year	Predicted number of locations	Actual number of locations
1998		
1999		

How do your predictions compare with the actual number of Starbucks locations that existed in 1998 and 1999? Refer to the comparison table you made earlier and record the actual values in the third column of the table above.

8. Did the logistic model provide reasonable predictions? Why is a logistic curve a better model for this data set than an exponential curve?

9. How does the logistic model account for the fact that the number of stores should eventually level off? To find out, click Zoom Out 〔🔍〕 several times to see how the curve behaves as the number of years increases. Estimate the level-off value for this function (called the *carrying capacity*). What does this value represent?

10. Save and print your TI InterActive!™ document.

Extensions

♦ Use the logistic modeling equation developed in this exercise to predict how many Starbucks shops there will be in the years 2000, 2005 and 2020. Describe any possible limitations the model might impose and under what circumstances it might fail completely.

♦ Can you think of some other real-world phenomena that might seem to behave exponentially but upon closer inspection actually grow (or decay) logistically? List as many examples as you can and, wherever possible, identify a reasonable level-off point or carrying capacity value.

Teacher Notes

Activity 10: Coffee Break

Activity Notes

♦ Note that this activity introduces logistic curves. Although the mechanics of the equation may be beyond the scope of your course, it is easy to explain how the shape of the curve is related to the way a company (or population) might grow.

♦ Most web sites are dynamic. Double-check the path to the company timeline before starting this activity with your class.

Sample Data

Year	Years Since 1986	Total Number of Starbucks Locations
1986	0	1
1987	1	17
1988	2	33
1989	3	55
1990	4	84
1991	5	116
1992	6	165
1993	7	272
1994	8	425
1995	9	676
1996	10	1015
1997	11	1412

Analysis and Questions - Key

1. $A = 1$.

2. $B = 2.05$.

3. For 1998, predicted = 5509, actual = 1886; for 1999, predicted = 11293, actual = 2200.

4. There is a big difference between the predicted values and the actual values because the modeling equation does not take into account that eventually, the number of stores will stop growing and start to level-off.

5. $regEQ(x) = (3972.15)/(1 + 434.8*(e)\wedge(-.498717\ x))$.

6. The logistic equation fits the data much better.

7. For 1998, predicted = 1896, actual = 1886. For 1999, predicted = 2386, actual = 2200.

8. The logistic model values match closely with the actual values. This model is more appropriate because it accounts for the fact that eventually the number of stores will reach a limiting value.

9. The level-off value is roughly 4000. This represents the expected value for the total number of Starbucks stores.