Cubic Inflection

by John F. Mahoney
Banneker Academic High School, Washington, DC
mahoneyj@sidwell.edu

Abstract: This activity is an applications of derivatives. It introduces students to an interesting property of cubics and a method of proving that property using the TI-89 scripts. They then use the symbolic capacity of their calculator to generalize upon specific results.

NCTM Principles and Standards:
Algebra standards

a) Understand patterns, relations, and functions
b) generalize patterns using explicitly defined and recursively defined functions;
c) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
d) use symbolic algebra to represent and explain mathematical relationships;
e) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
f) draw reasonable conclusions about a situation being modeled.

Problem Solving Standard build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof Standard

a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;
d) select and use various types of reasoning and methods of proof.

Key topic: Applications of derivatives. Minima, maxima, and inflection points. Scripts, formal proofs.

Degree of Difficulty: Medium
Needed Materials: TI-89 calculator
Situation: Cubic polynomials have many interesting properties. In this activity we'll use calculus to investigate one of them with the aid of the TI-89 calculator. Consider a line containing the relative minimum and relative maximum of a cubic. Where else does this line cross the cubic? What is the midpoint of the line segment between the relative minimum and relative maximum of the cubic?

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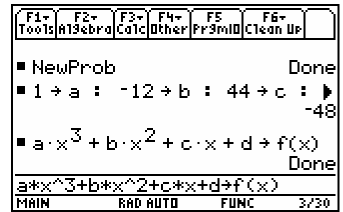
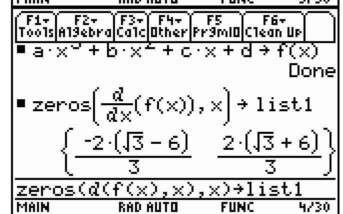
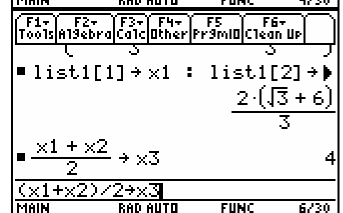
Choose arbitrary values for the coefficients of a cubic polynomial and store the result in f(x).



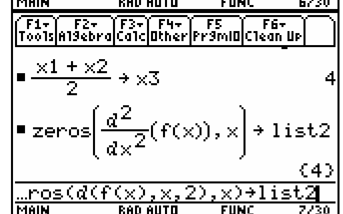
Find the zeros of the first derivative and store the result in list1. If there are no real zeros, go back and change the coefficients of the cubic.



Store these zeros as x1 and x2 and store their average as x3:

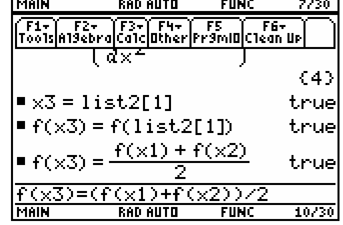


Now compute the zeros of the second derivative of f(x) and store that in list2:

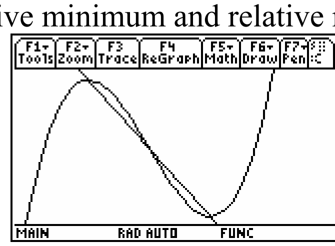


Now compare x3 and list2[1] and f(x3) with f(list2[1]) and with

$$\frac{f(x1)+f(x2)}{2}$$



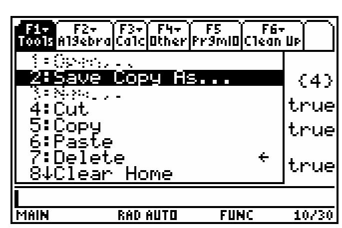
What have we shown? We've shown for this cubic that the midpoint of the line segment connecting the relative minimum and relative maximum of a cubic is the



inflection point of that cubic!

Is this property always true? You could scroll back and change the coefficients of the cubic and execute the steps again, but we'll take a different tack. We'll turn what we've written into a script which can be followed for any cubic:

Press F1 and choose "Save Copy As"



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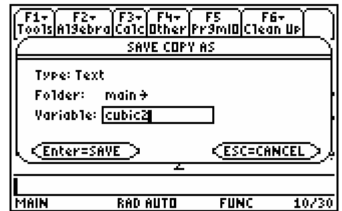
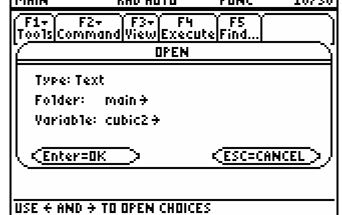
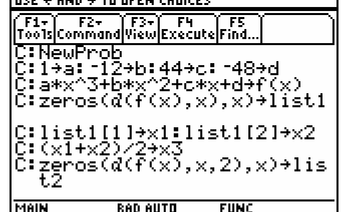
I named this script “cubic2”



From the application menu [APPS] choose 8:Text Editor:

and open the script named cubic2:

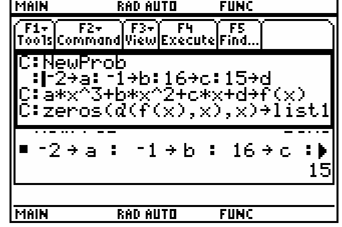




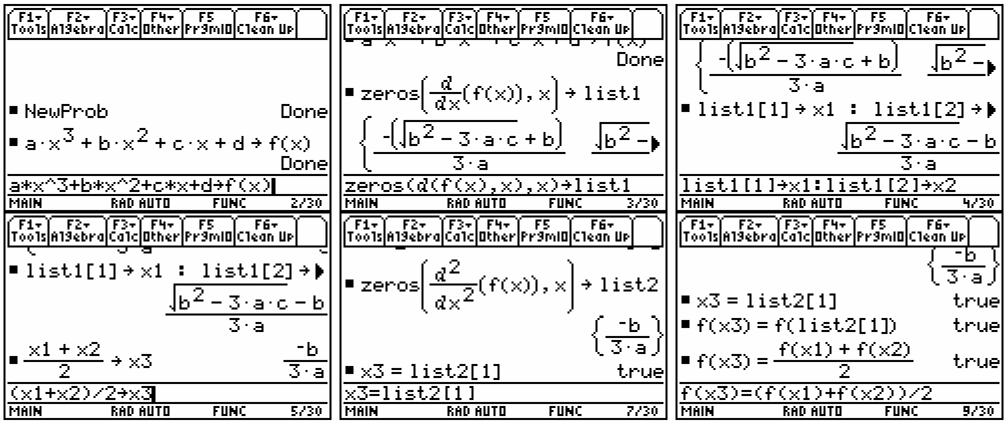
This script contains the commands that were originally typed in. We can play this script for other choices of a, b, c, and d: First press [F3]to change the view to A: Script View

Pressing [F4] executes each line. Change the values of a, b, c, and d and continue pressing [F4] to see that the property is true with these choices.

Is it always true? Go back to the line “C: -2→a: -1→b: 16→c: 15→d” and choose 4:Clear Command from the [F2] Command menu which will erase the C: in the line “C: -2→a: -1→b: 16→c: 15→d”.



Now run the script again and observe that the calculator creates a proof of this property:



Scripts can be very useful in proving properties. Here we have shown that the x- and y-coordinates of the inflection point of a cubic are the average of the x- and y-coordinates of its critical points. Note that the x-coordinate of the inflection point is -b/(3a). This is very similar in form to the formula for the x-coordinate of the vertex of a parabola in standard form! Can you extend this result?

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