# Let's develop your Quads!

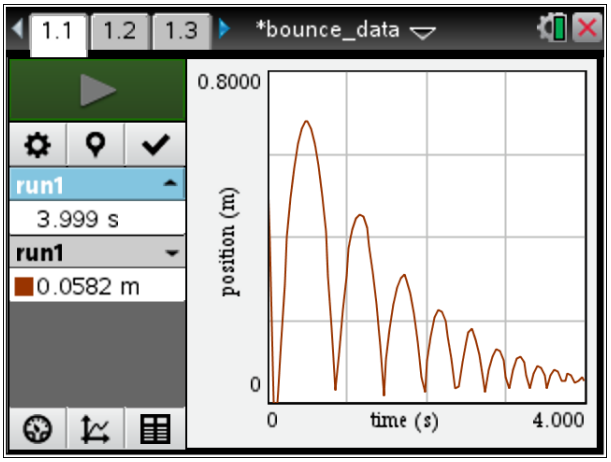
**Modelling Quadratics using a motion detector with a Graphics Calculator**

**Instructions:**
This task investigates the mathematics of a bouncing ball. You will collect the data using your TI-Nspire and a motion detector. (Every student is to have their own set of data).

Connect the CBR2 to the TI-Nspire handheld (or computer) using the provided cables.

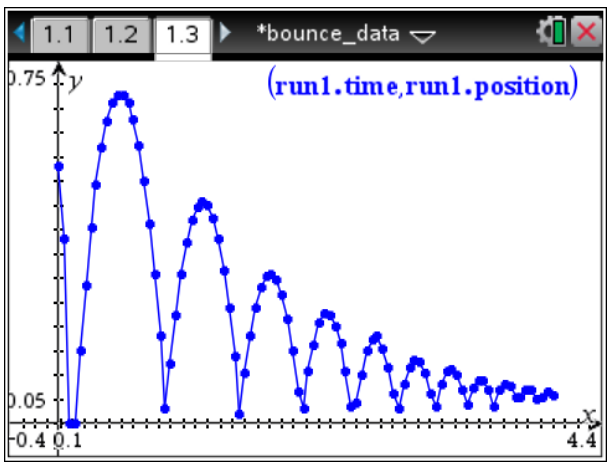
Hold the motion detector about 0.5 m above the ball (*use a small racquet ball but a basketball will work OK*) and drop the ball onto a smooth, level surface (*Do not bounce the ball!*) You can follow the ball if it moves sideways but keep the motion detector at a constant height. When you drop the ball press TRIGGER on the CBR2. The plot will appear opposite to the actual measured distance as the **Reverse Readings** option was chosen in the collection setup.

**Data:**
The resulting plot of ***distance*** (p***osition***) vs **time** should appear to be a series of parabolic sections with decreasing maximum heights similar to what is shown here. You need to have a plot that gives you at least 5 peaks. The entire set of data is in lists **Time** and **Position**.




If you are **<u>not happy</u>** with your results, press the to start a new collection.

Whilst a lot of analysis can be done within the **Vernier DataQuest** application the data will be plotted in the more familiar **Graphs** application for mathematical analysis in this task.

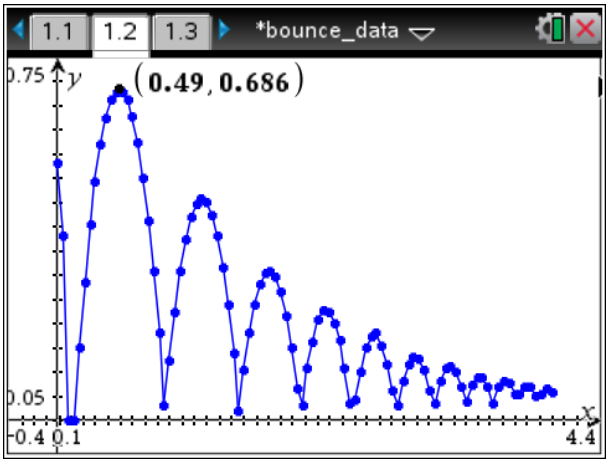



Open a **Graphs** application, setup a **Scatterplot** (**Graph Entry/Edit**). Use the [var] key to enter the variables ***time*** and ***position***. ***(run1.time*** & ***run1.position***). Use the **Window/Zoom>Zoom-Data** menu to rescale.

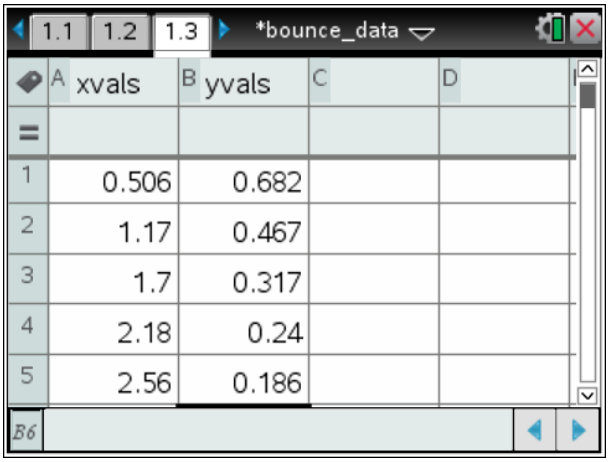
Use **Geometry>Points & Lines>Point** to place points on the peaks. If **Point On** appears press [tab] and place the **Point** at the peak. Right click ([ctrl] [menu]) on the point and display the coordinates. Record in the table. Drag the point to obtain the coordinates of the first five peaks.

Put your bounce peak coordinates in the table below

| Bounce no. | X (time) | Y (height) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

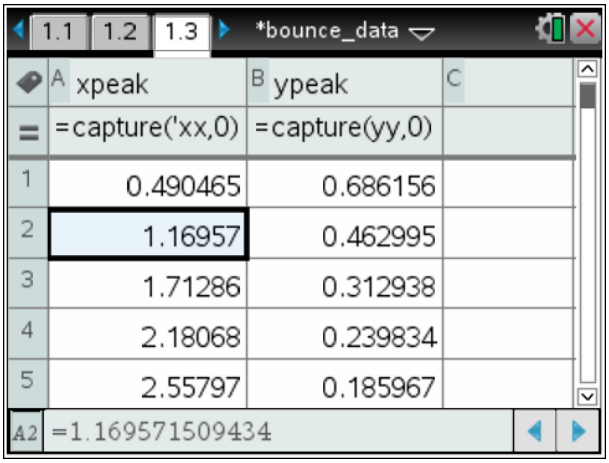



Enter the x and y values in the L&S.

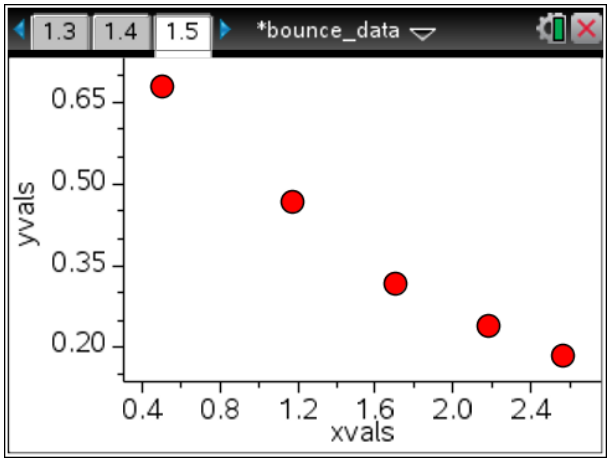



Note: it is also possible to use manual data capture (not available on iPad App) to place all the values in the Lists & Spreadsheet application. Identify the first peak with coordinates. Right click (ctrl menu) on the x coordinate and store as ***xx***. Repeat for the y-coordinate and store as ***yy***.
Set up the manual Data Capture (**Data>Data Capture>Manual**) in the L&S. Return to the **Graphs** page. Press ctrl . [capture] to capture the first point. Drag the point to the next peak and press ctrl . to capture this new point. Repeat for a total of 5 peaks.

Note: the list names xpeak and ypeak are arbitrary and were chose to be different to the earlier example done manually)



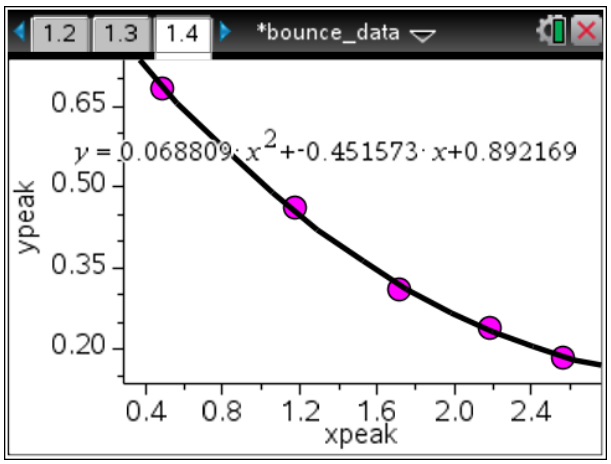

Open a **Data & Statistics** page and plot ***ypeak*** against ***xpeak*** (or ***xvals*** against ***yvals*** if not using the data capture method).



**Discussion & Questions:**

**In this task you will find a quadratic function that will most closely join the peaks of the bounces.**

1. The TI-Nspire software has a built-in function that allows it to compute the best-fitting quadratic equation for a set of data that appears to be a parabola. This is called a quadratic regression.




Use **Analyze>Regression>Show Quadratic** to obtain the model equation.

Note: it is often mistakenly assumed that the plot is an exponential decay. This is not the case although the bounce heights are declining in a set ratio (geometric sequence) but the time interval is not consistent!

If you plot bounce height against bounce number then this will be an exponential model.

2. Use the peak data from any **3** bounces from the table to **construct** three simultaneous equations that would allow you to find the values of a, b & c in the quadratic of form $y = ax^2 + bx + c$

(*It is possible for students to generate unusual. i.e. negative quadratics using the minimal number of data points based on experimental data so be prepared*)

$$\text{solve}\left(\begin{cases} 0.682=a\cdot(0.506)^2+b\cdot 0.506+c \\ 0.317=a\cdot(1.7)^2+b\cdot 1.7+c \\ 0.186=a\cdot(2.56)^2+b\cdot 2.56+c \end{cases},a,b,c\right)$$

$$a=0.074669 \text{ and } b=-0.470414 \text{ and } c=0.900912$$

3. Use the matrix function on your calculator, or algebraic methods, to **determine** the values a, b & c.

Hint: $y = ax^2 + bx^1 + cx^0$ (what does $x^0$ equal?)

e.g. using sample data points (0.506, 0.682), (1.7, 0.317) & (2.56, 0.186)

$$\begin{bmatrix} .506^2 & .506^1 & .506^0 & .682 \\ 1.7^2 & 1.7^1 & 1.7^0 & .317 \\ 2.56^2 & 2.56^1 & 2.56^0 & .186 \end{bmatrix}$$

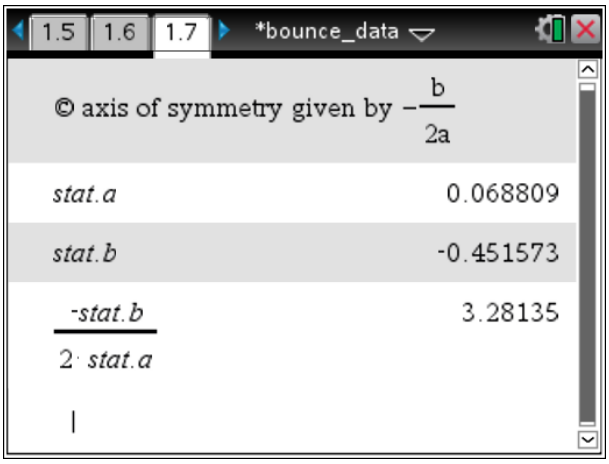
(*You may prefer to use the inverse matrix method instead of the Row Reduced Echelon Form method*.)

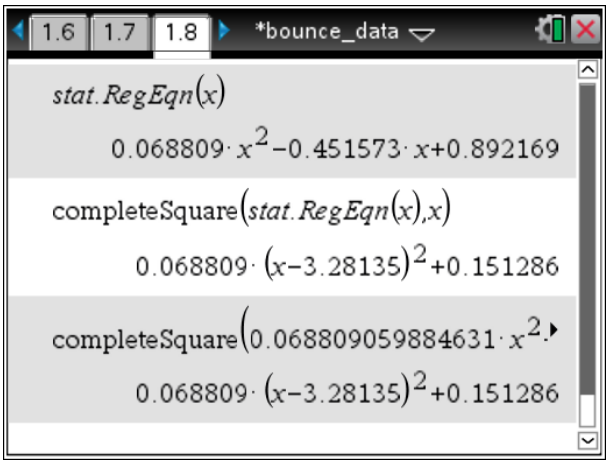
$$\text{rref}\left(\begin{bmatrix} (0.506)^2 & (0.506)^1 & (0.506)^0 & 0.682 \\ (1.7)^2 & (1.7)^1 & (1.7)^0 & 0.317 \\ (2.56)^2 & (2.56)^1 & (2.56)^0 & 0.186 \end{bmatrix}\right)$$

$$\begin{bmatrix} 1. & 0. & 0. & 0.074669 \\ 0. & 1. & 0. & -0.470414 \\ 0. & 0. & 1 & 0.900912 \end{bmatrix}$$

4. All equations determined so far will not be exactly the same but both should fit the data quite well. Compare the equation you obtained in Question 1 with that obtained in Question 3. Suggest some reasons why they are similar but different.

5. From the equation obtained in Question 1, use an appropriate mathematical technique to find the <u>axis of symmetry</u> of the quadratic. What significance does the axis of symmetry (*x*–value) have in this model? Use the equation to determine the height of the ball bounce at the axis of symmetry. Interpret your findings with reference to the practical situation.




6. It has been mathematically determined that a quadratic model is in fact a very good fit to this practical situation. Why is the standard quadratic model, where *x* can have any value (i.e. a domain of R), not suitable for this practical situation? What is a suitable restriction for the *x* values in the model you have found? Give reasons why the restriction you have selected is suitable.

7. The equation you found in Question 1 can be expressed in the form $y = a(x - h)^2 + k$. Show full algebraic working to convert your equation to the form $y = a(x - h)^2 + k$. Comment on the significance of the h & k values in the practical (real-life) situation of the experiment by comparing your values obtained here with those determined in Question 5.