



Exercises

1. State the product rule for a function of the form $u(x) \cdot v(x)$.

2. Apply the product rule to the function $\sin(x) \cdot \ln(x)$.

3. Do you agree or disagree with the following statement? Explain.

$$\int \frac{d}{dx}(f(x)) dx = \frac{d}{dx} \left(\int f(x) dx \right) = f(x)$$

4. What is the integral of the left side of the product rule?

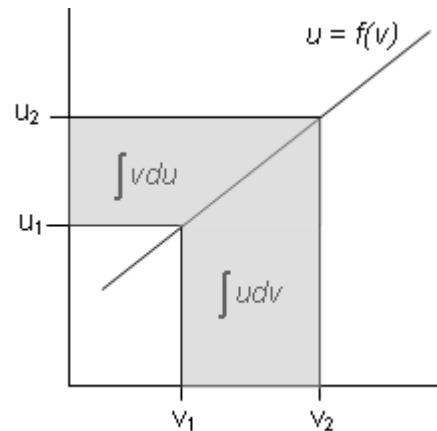
$$\int \left(\frac{d}{dx}(u(x) \cdot v(x)) \right) dx =$$

5. What is the integral of the right side?

$$\int \left(u(x) \cdot \frac{dv}{dx} + v(x) \cdot \frac{du}{dx} \right) dx =$$

6. Explain the relationship between the areas shown on the graph and the following equation:

$$\int_{v_1}^{v_2} u \cdot dv = u \cdot v - \int_{u_1}^{u_2} v \cdot du$$





Exploring the Area Under a Curve

7. Use the method of **integration by parts** to compute the integral of **ln(x)**.

Remember the formula for Integration by parts is $\int u \cdot dv = u \cdot v - \int v \cdot du$

$$\int \ln(x) \cdot 1 \, dx \rightarrow u = \ln(x) \text{ and } dv = 1 \, dx$$

$$du = \quad \quad \quad v =$$

Result =

Check by integration directly. (**Home > F3:Calc > 2:Integrate**) or (**Home > 2nd 7**)

Consider the function $f(x) = \sin(\ln(x))$.

$$u = \sin(\ln(x)) \rightarrow du = \frac{\cos(\ln(x))}{x} dx$$

$$dv = dx \rightarrow v = x (+C)$$

$$\begin{aligned} \int \sin(\ln(x)) \cdot 1 \, dx &= x \cdot \sin(\ln(x)) - \int x \cdot \frac{\cos(\ln(x))}{x} dx (+C) \\ &= x \cdot \sin(\ln(x)) - \int \cos(\ln(x)) dx (+C) \end{aligned}$$

8. Find $\int \cos(\ln(x)) dx$.

$$u = \quad \quad \quad du = \quad \quad \quad dv = \quad \quad \quad v =$$

$$\int \cos(\ln(x)) dx =$$

9. Substitute the result for **cos(ln(x))** into the result for **sin(ln(x))**.

$$u = \quad \quad \quad du = \quad \quad \quad dv = \quad \quad \quad v =$$

$$\int \sin(\ln(x)) dx =$$

10. Use integration by parts to solve the following. If you need to use integration by parts more than once, do so. Check your result.

a. $\int \tan^{-1}(x) \, dx$

b. $\int x^2 \cdot e^x dx$

c. $\int x \cdot \tan^{-1}(x) \, dx$

d. $\int x \cdot \cos(2x + 1) \, dx$

11. (**Extension 1**) Does it matter in which order **u(x)** and **v(x)** are selected for the method of integration by parts?
12. (**Extension 2**) Is there likely to be an integration rule based upon the quotient rule just as Integration by Parts was based upon the product rule?