# Quartic Regions

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**Abstract:** This activity is an  applications of derivatives and of definite integrals.  It introduces students to an interesting property of Quartics which students can verify using the symbolic capacity of their calculators.

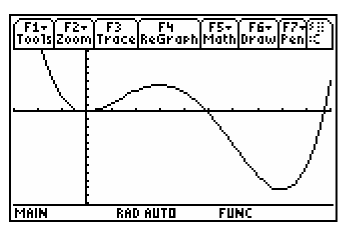
**NCTM Principles and Standards:**
**Algebra standards**
a) Understand patterns, relations, and functions
b) generalize patterns using explicitly defined and recursively defined functions;
c) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
d) use symbolic algebra to represent and explain mathematical relationships;
e) judge the meaning, utility, and reasonableness  of the results of symbol manipulations, including those carried out by technology.
f) draw reasonable conclusions about a situation being modeled.

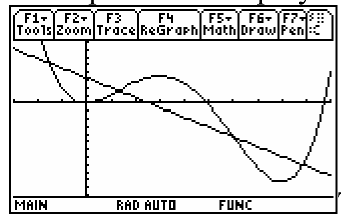
**Problem Solving Standard** build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

**Reasoning and Proof Standard**
a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;

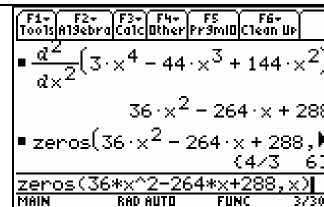
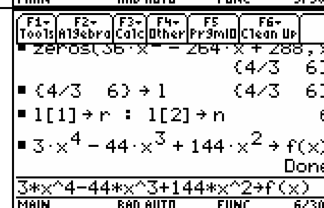
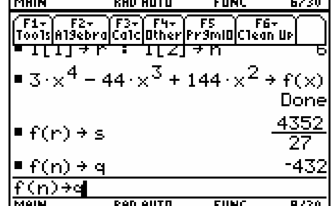
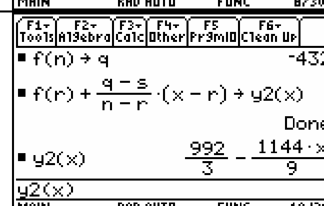
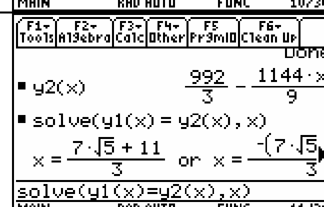
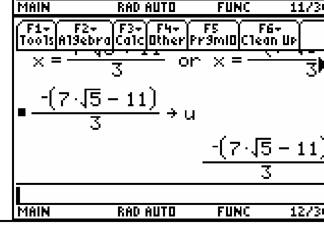
**Key topic:** Applications of derivatives. Inflection points. Application of Definite Integrals - area between curves.

**Degree of Difficulty:** Advanced
**Needed Materials**: TI-89 calculator
**Situation:** Quartic polynomials have many interesting properties. The graphs of most fourth degree polynomials have "three bumps".  Which of the bumps is the largest.  In this activity we'll use calculus to investigate this question with the aid of the TI-89 calculator.

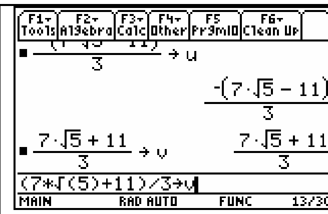



Consider the graph of $f(x) = 3x^4 - 44x^3 + 144x^2$ whose three bumps are:  a relative minimum at x = 0, a relative maximum at x = 3, and another relative minimum at x = 8. I would like to suggest one way of measuring the size of these bumps is to draw the line passing through the two inflection points of the polynomial and find the area between each of the bumps and this line. 

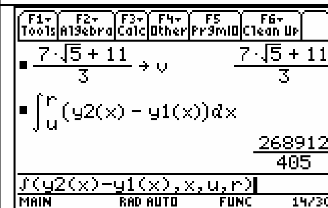
The TI-89 can perform the calculations with ease:

| | |
|---|---|
| The zeros of the second derivative give us the x-coordinates of the points of inflection: 4/3 and 6 |   |
| Now we find the y-coordinates of the points of inflection |     |
| We can find the equation of what I'll call the "inflection line", y2(x), which passes through the two inflection points of the original function |   |
| The inflection line intersects the graph of the original polynomial in four points. We are interested in the left most and right most points whose x-coordinates are: $x = \frac{7\sqrt{5}+11}{3}$ *and* $x = \frac{-(7\sqrt{5}-11)}{3}$ |     |

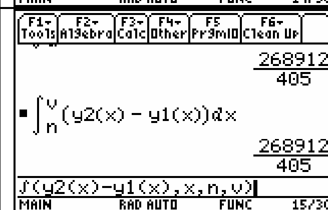
 



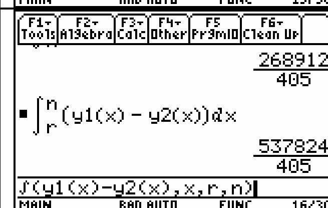

The area of the left "bump" is $\frac{268912}{405}$

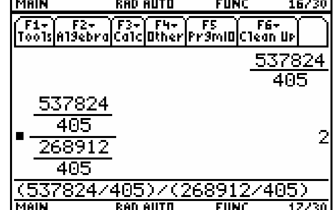


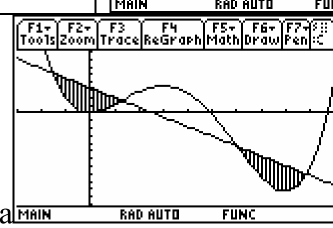

And, would you believe it? The area of the right "bump" is also exactly $\frac{268912}{405}$



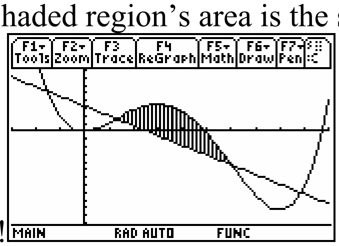

Finally - take a look at the area of the middle "bump" $\frac{537824}{405}$. This is exactly twice the areas of each of the other two bumps and therefore it is the sum of the other two as well.




Conclusion: These two shaded regions have the same area
This shaded region's area is the sum of the two in the previous graph!




Try this with your own fourth degree polynomial!