



Problem – Modeling tree growth

To model the tree's growth for the years 2002 to 2008:

- Draw a short vertical line segment starting at the x -axis.
- Measure the length of the segment. Place the measurement at the top of the segment.
- Drag the grid down to view larger y -values.
- Extend the line segment to the appropriate length. First, make the segment as close to the correct length as you can. Then, adjust the window settings to view a small range of y -values on either side of the value you want. For example, if you are trying to make a line 64 units long, adjust **YMin** to 54 and **YMax** to 74. This will make the **YScale** equal to 2, and you can end your segment on a grid point.
- Adjust the window settings as necessary. The x range should start just left of $x = 0$ and go to just right of $x = 10$. A **YMin** of -30 is good to see the labels on the axes. As you work backwards, you will need to increase the **YMax** to see the tops of all of the segments.
- You will notice that all of the lengths are powers of two. Complete the table below.

Year	$x =$	Height (feet)	Power of Two
2001	1	2	2^2
2002	2	4	2^4
2003			
2004			
2005			
2006			
2007			
2008			

Discussion Question

- If $x = 1$ represents the year 2001, what represents 2000? 1999? 1995?



To graph the sequence $a_n = 2^n$ for $n = 1..8$:

- Enter the values of n in Column A on page 1.6. You can type the values 1...8 by hand or use the **seq(** command. (**seq(x, x, 1, 8)**).
- Go to the formula bar of Column B on page 1.6 and type **2^n**. Choose Variable (not Column reference) when prompted. The handheld lists the numbers $2^1...2^8$, the values of $a_n = 2^n$ for $n = 1...8$.
- Return to page 1.4 and change the **Graph Type** to **Scatter Plot**.
- Press **(ctrl) + [G]** to display the Scatter Plot Entry Line.
- Choose **n** for the **xlist** and **sequence** for the **ylist**.
- Press **(ctrl) + [G]** to hide the Scatter Plot Entry Line. The sequence appears as a set of white dots.

To model the tree's growth for the years 1992 to 2000:

Work backwards through time, cutting the tree in half each time you step back a year.

- Draw a vertical ray starting at the x-axis.
- Use the **Measurement Transfer** tool to transfer the length of the tree you want to cut in half (the tree from the following year) onto this ray. For example, if you are making the 2000 tree, transfer the length from 2001 tree by clicking on its length measurement, then on the ray.
- Use the **Midpoint** tool to cut the tree in half. Click on the point on the x-axis, then the point created by the measurement transfer.
- Draw a segment from the point on the x-axis to the midpoint. This is the tree.
- Hide the ray and the two extra points used to create the segment.
- Measure the length of the segment. Place the measurement at the top of the segment. Record the measurement in the table.
- As you work backwards, you will need to shrink the y range to get a close-up view of the segments.
- These lengths are also powers of two! Complete the patterns in the right two columns of table below to see how.

Year	$x =$	Height (feet) Decimal form	Height (feet) Fraction form	Power of Two
2000	0	1	1	2^0
1999	-1	0.5	$\frac{1}{2}$	2^{-1}
1998				
1997				
1996				
1995				
1994				
1993				
1992				

Discussion Questions

- Adjust the window so that all the segments are visible. Observe the shape of the curve.
- What do you notice about the denominators of the fractions in this sequence?

To extend the sequence $a_n = 2^n$ to $n = -8..8$:

- Column A on page 1.6 already lists the numbers 1...8, the values of n . You want to extend this list so that it lists all the values of n from -8...8. You can type these values by hand or use the **seq** command. (**seq(x, x, -8, 8)**).
- The sequence in column B and the scatter plot on page 1.4 update automatically.
- You may need to adjust window settings to see the scatter plot.
- Write a rule for evaluating negative powers of 2.