## Math Objectives

- Students will recognize that a *p*-value only has meaning if the null hypothesis is true (a conditional probability).
- Students will interpret a *p*-value in given contexts as the relative frequency for sample statistical values at least as extreme as that from the observed sample, assuming that the null hypothesis is true.
- Students will recognize the relationship between sample size and *p*-values: in general, for sample means giving *p*-values less than 0.5, increasing the sample size decreases the *p*-value.

## Vocabulary

- alpha value
- population
- relative frequency
- sample mean
- sample size
- sample standard deviation
- sampling distribution

## About the Lesson

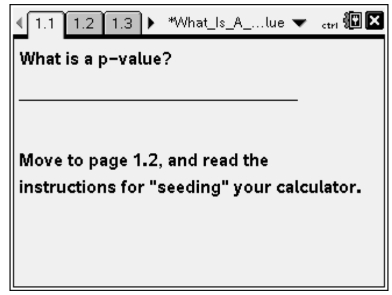
Students begin with a null hypothesis specifying the mean of a normally distributed population with a given standard deviation.

- Students generate an observed outcome (a sample of fixed size), which determines a *p*-value. That value is represented by a shaded region in the sampling distribution.
- Students then generate additional samples of the same size from the hypothesized population and observe where the means of these samples fall.
- From the graph of the simulated sampling distribution of sample means that were generated, students estimate the likelihood of getting by chance an outcome at least as extreme as the original observed sample mean if the hypothesis is true.

## Prerequisites

Students should have knowledge of random sampling and of sampling distributions.

The students are asked to compare *p*-values and alpha levels. If they do not have experience with alpha values, Question 9 should be omitted.




**TI-Nspire™ Technology Skills:**

- Download a TI-Nspire document
- Open a document
- Move between page
- Seeding a random number

**Tech Tips:**

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

**Lesson Materials:**

*Student Activity*

- What_Is_A_P-value.PDF
- What_Is_A_P-value.DOC

*TI-Nspire document*

- What_Is_A_P-value.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

## Related Statistics Nspired Activities

Sampling Distributions

The Meaning of Alpha

## TI-Nspire™ Navigator™

- Use Screen Capture to compare student results.
- Send tns File to students.
- Use Quick Poll to determine student understanding.

**Teacher Note**: It is highly recommended that students work through *The Meaning of Alpha* prior to this lesson.

## Discussion Points and Possible Answers

**Tech Tip:** Page 1.2 gives instructions on how to seed the random number generator on the handheld. Page 1.3 is a *Calculator* page for the seeding process. Ensuring that students carry out this step will prevent students from generating identical data. (Syntax: RandSeed #, where # is a number unique to each student.)

**Teacher Tip:** Once students have seeded their random number generators, they do not have to do it again unless they have cleared all of the memory. But it is important that this be done if the memory has been cleared or with a new device, as otherwise the "random" numbers will all be the same as those on other similarly cleared devices.

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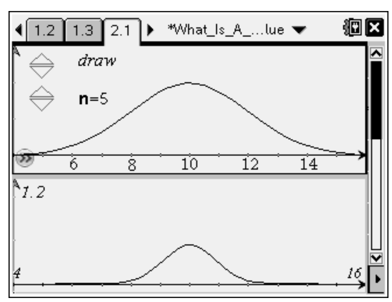
**Move to page 2.1.**




Consider the following hypothesis test scenario:

$H_0$: $\mu = 10$ and $H_a$: $\mu > 10$

The top screen of page 2.1 of the tns file represents a population whose mean is 10, as assumed in the null hypothesis.

**Teacher Tip:** The lesson assumes $H_a$: $\mu > 10$ but could be adapted for $H_a$: $\mu < 10$. You might want to have students suggest a range for the horizontal axis, based on their knowledge of the empirical rule—68, 95, 99.7—that nearly all of the values lie within three standard deviations of the mean for a normal distribution, so 4 to 16 is an appropriate window, as indicated in the lower screen.

**Teacher Tip:** After selecting a sample, student may have to click in an empty white space in order to activate the shading in the lower screen.

1. Use the up arrow (▲) to draw a random sample of size 5 ($n$ = 5) from this population.

   a. What do the dots in the top graph represent?

   **Sample Answer:** The values in the top graph are elements of the population that were selected in the sample.

   b. What does the white dot in the bottom graph represent?

   **Sample Answer:** The white dot in the bottom graph is the mean of the sample shown on the top graph.

   c. Estimate the values of the elements in the sample and give the sample mean.

   **Sample Answer:** Answers will vary depending on the sample that is chosen: for example: Sample = {9.3, 10.1, 10.9, 12.6, 13.1}; sample mean = 11.2.

**Teacher Tip:** Students may not remember that areas in distributions are equivalent to probabilities that the variable falls within that region on a random draw. Question 2 below revisits this concept. It is important that students recognize and are able to articulate this notion before going on to Question 3. If you have Navigator, a Quick Poll would be useful in making sure students understand this concept.

2. a. Considering the alternative hypothesis stated above, does it seem likely that your sample came from the hypothesized population or from a population whose mean is considerably larger than 10? Explain your reasoning.

   **Sample Answer:** Students with sample values far to the right might respond that it seems as if their sample came from a population whose mean is larger than 10. Students with sample values clustered around 10 are likely to believe that their sample could have come from the hypothesized population.

**TI-Nspire Navigator Opportunity:** ***Screen Capture:***
**See Note 1 at the end of this lesson.**

   b. Estimate the probability that future sample means will fall to the right of this sample mean.

   **Sample Answer:** Answers will vary depending on the observed sample means, but students may use general statements such as “really likely” if, for example, the sample mean is below 10 or “very unlikely” if it is around 12 or 13.

Use the random sample you have for Question 1 to answer the next set of questions.

The term *p-value* is the probability that you would get a sample mean at least as extreme in the direction of the alternative hypothesis as the one from the random sample you drew if the null hypothesis is correct. The shaded area in the lower screen indicates the *p*-value.

**Teacher Tip:** Students should keep the sample they have for this problem constant for the next set of questions. They will come back and play with different samples later in the activity.

**Teacher Tip:** This activity does not deal with the calculation of the *p*-value but focuses on the interpretation of what a *p*-value actually represents. Standard texts will describe how to do the related calculations and how to account for knowledge about the standard deviation of the population. The concept of *p*-value developed in the activity applies to proportions as well as to sample means.

3. a. Explain why this area represents a probability.

**Sample Answer:** The relative frequency of elements selected from a population falling between any two values, *a* and *b*, is equal to the area under the density curve between these points. Thus, $P(a < x < b)$ = area between *a* and *b* under the curve.

b. Interpret the *p*-value you have on your screen.

**Sample Answer:** Answers will vary. For a sample mean of 11.2, the *p*-value is about 0.09, so 9% of any future samples from a population with an assumed mean of 10 will have means of 11.2 or greater.

c. Based on your observed *p*-value, does it seem likely that your sample came from a population with mean = 10? Why or why not?

**Sample Answer:** Answers will vary. Students may suggest that their *p*-value was very small, and so it seems likely that their observed sample mean corresponding to that *p*-value would be unlikely to occur by chance in that population; or they may have a *p*-value that was large (i.e., 0.45), and so it seems very likely that their sample mean would occur in a sample from the hypothesized population.

**Teacher Tip:** Discuss student answers to Question 3 part c. The main idea behind deciding whether a sample could have come from the given population is to decide whether that sample is "typical" or "unusual." Focus on student responses that indicate this line of reasoning. The remainder of this activity is designed to help students see that low *p*-values indicate unusual samples.

**TI-Nspire Navigator Opportunity: *Screen Capture***
**See Note 2 at the end of the lesson.**

**Move to 2.2, without changing the *p*-value you found above.**

4. a. Without actually drawing additional samples, describe the sampling distribution of the sample means from the samples you would expect to get.

   **Sample Answer:** Student answers will vary. They should recognize that if the null hypothesis, $H_o$: $\mu = 10$, is correct, then the sample means should center around 10 and interpret them in terms of their observed *p*-value; for example, if the *p*-value is 0.09, about 9% of the sample means should be greater than their observed outcome of 11.2.

   b. Now use the up arrow (▲) to draw more samples of size 5 from the given population. Explain what the values plotted on the horizontal axis represent.

   **Sample Answer:** Each time you press draw, you generate five new samples, and the means of those samples are the values that are appearing on the horizontal axis.

   **Teacher Tip:** Be sure students recognize that the values generated are not the sample values as on page 2.1. Instead, the means of the samples are plotted, with five sample means displayed for each click.

   c. How does the simulated sampling distribution of the sample means compare to your predictions in Question 4 part a? Explain any differences.

   **Sample Answer:** Student answers will vary. They will likely get a distribution somewhat similar to the one described in Question 4 part a. Any differences can be explained by chance, or if the distribution does not seem to center around 10, it could suggest that the evidence is not supporting the null hypothesis that the mean of the population is 10.

d. From your simulation of sample means, estimate the likelihood of getting a sample from the given population so that the new sample mean is at least as extreme as the original observed sample mean.

**Sample Answer:** Student answers will depend on their simulated distribution of the sample means. Ideally, student answers will be their displayed *p*-values.

> **Teacher Tip:** Be sure students recognize that it is not possible in real applications of statistics to draw more than one sample. But to decide whether an individual sample is likely or unlikely to have come from the hypothesized population, it is necessary to examine many different samples from that population.

5. Use ctrl tab to move to the line below the screen with the graph. Type **reset()** in the box and press enter. (These instructions are also on **page 2.3** of the tns file.) Resetting will allow you to simulate another sampling distribution of sample means from samples of size 5.

> **Teacher Tip:** If students do not reset, when they clear the screen by pushing the down arrow ( ▼ ) and redraw a new set of 100 samples, the same distribution they just had will disappear. Resetting is necessary to generate new random samples.

a. Before generating new samples, predict how the new distribution of sample means will compare to the one you generated for Question 4.

**Sample Answer:** Students may predict a distribution similar to the one they found in Question 4, or they may recognize the possible variability and describe a very different sample.

b. Remember that the vertical line labeled $\bar{x}$ represents your original sample mean. Predict the proportion of future samples that will have means to the right of that sample mean.

**Sample Answer:** Students should predict a proportion about equal to the displayed *p*-value.

c. Generate 100 sample means. How well does their distribution match your predictions in parts a and b? Explain any differences.

**Sample Answer:** In most cases, the results should be fairly similar. The distribution should be centered about 10 and symmetric. The proportion of points falling in the shaded region should be about equal to the *p*-value. Any differences could be explained by chance.

**Teacher Tip:** You may want students to generate several more simulated distributions by using the reset command and discuss as a class what they are finding.

**TI-Nspire Navigator Opportunity:** ***Screen Capture***
**See Note 3 at the end of the lesson..**

**Teacher Tip:** You might also want students to return to page 2.1 in the tns file and generate another "original" sample mean for $n = 5$ before you go to Question 6 and have them answer questions in Question 5 again, contrasting their answers with the ones they had the first time through. This is also a good opportunity for students to share their results and talk about any general patterns they observe.

**Return to page 2.1.**

6. Change the sample size to 10 ($n = 10$). Draw another random sample.

a. What are the sample mean and the corresponding *p*-value for this random sample?

**Sample Answer:** "My sample mean was 9.96, and the *p*-value was 0.52."

b. Describe what your *p*-value means.

**Sample Answer:** If the null hypothesis is true, i.e., the mean of the population is 10, then a *p*-value of 0.52 would indicate the probability of getting a sample mean at least as large as 9.96 by chance. In other words, about half of all samples would have a sample mean greater than or equal to 9.96 if the population mean is 10.

c. Compare your *p*-value with that of other students in your class. Why do these values differ?

**Sample Answer:** They differ because each *p*-value comes from a particular sample, and different students had different samples.

d. Sketch the simulated sampling distribution of sample means you might expect to get if you were able to draw 100 samples rather than just one.

**Sample Answer:** The sketch should show a distribution that is relatively symmetric around a mean of 10, with a smaller spread than the sampling distribution for samples of size 5. Students may not recognize how spread is affected by sample size without more experience changing the sample size.

> **Teacher Tip:** The activity *Sampling Distributions* provides experience with this idea as well.

e. Predict the proportion of future samples that will have means to the right of your displayed sample mean.

**Sample Answer:** Students should predict a proportion about equal to the displayed *p*-value.

**Move to page 2.2, using your new *p*-value.**

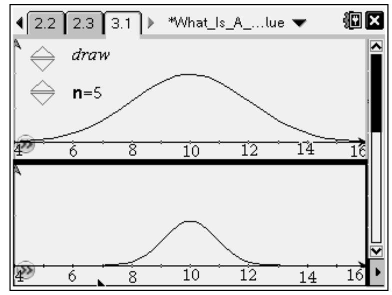
7. Draw 100 samples of size $n = 10$ and compare this simulated sampling distribution of sample means to your prediction in Question 6 part c and d.

**Sample Answer:** The distribution should be relatively normal, symmetric around 10, with the sample means clustered closer to 10 than was true for samples of size 5. The proportion of means in the shaded region should be about equal to the *p*-value.

> **Teacher Tip:** Have students compare their *p*-values and graphs from Question 7. Be sure students understand that *p*-values measure the proportion of samples from the given population whose means are at least as extreme in the direction of the alternative hypothesis as the mean of the observed sample.

**TI-Nspire Navigator Opportunity: *Screen Capture***
**See Note 4 at the end of the lesson.**

**Move to page 3.1.**




8. Assume $H_0$: $\mu = 10$ and Ha: $\mu > 10$.
   a. Draw samples from the population for $n = 5$ until you find a *p*-value less than 0.1. What are your sample mean and the corresponding *p*-value?

   **Sample Answer:** One answer might be for an observed sample mean of 11.5, the *p*-value for $n = 5$ is about 0.05.

   b. Change the sample size to $n = 10$, and draw samples until you get a sample mean close to the sample mean you found in part a. What is the corresponding *p*-value? Repeat the process for sample size $n = 15$.

   > **Teacher Tip:** Be sure students are looking for the "same" sample mean, not the "same" *p*-value.

   **Sample Answer:** A sample answer might be: for an observed sample mean of 11.6, when $n = 10$, the *p*-value is about 0.01; for $n = 15$, a sample mean of 11.5 gives a *p*-value of about 0.00, which means the value is very close to 0.

   > **Teacher Tip:** The tns file permits only 100 samples to be selected using the "draw" slider. If students reach a point where they can no longer draw samples but need more, they should click the down arrow (▼) to continue.

   c. Jordan found an observed sample mean for a sample size of $n = 5$ and claimed that a sample size of $n = 20$ with the same sample mean would have the same *p*-value. Explain whether you agree with Jordan and why.

   **Sample Answer:** The *p*-values would not be the same. For $n = 20$, the *p*-value would be much smaller. As the sample size increases, the standard deviation of the sampling distribution decreases, and the curve becomes taller and narrower. Thus, a given value for a sample mean will have less area beyond that sample mean as the sample size increases.

9. All of the work in this activity has assumed that $H_0$: $\mu = 10$ and $H_a$: $\mu > 10$. How would your thinking change if $H_a$: $\mu < 10$?

**Sample Answer:** The only difference would be the shaded region determined by the *p*-value would be in the lower tail of the sampling distribution of the sample means. All of the reasoning about what a *p*-value represents would be the same. In particular, note that the words "at least as extreme" could apply to values either less than or greater than a given sample mean.

**Teacher Tip:** You may want students to consider how to interpret two-sided *p*-values at this point as well.

10. In earlier work, you studied alpha levels. Describe similarities and differences between alpha levels and *p*-values.

**Sample Answer:** Both alpha levels and *p*-values are probabilities that future sample means would fall beyond a given point. Both begin by assuming the null hypothesis is true.

An alpha level is the relative frequency for sample statistic values that lead to a “reject the null” conclusion based on a fixed rejection criterion selected by the researcher before a sample is drawn, given that the null hypothesis is actually true.

The *p*-value is based on the observed statistic—in this case, the sample mean—from the random sample and is the probability that a future sample has a mean at least as extreme as the one from the random sample you drew, if the null hypothesis is correct. The observed sample mean marks the boundary of the shaded region, and the *p*-value describes the area of the shaded region.

If your sample mean is in the region denoted by the alpha level, you would consider that to be evidence that the observed outcome (the sample mean in our work) was unlikely to occur by chance for that alpha and would reject the null hypothesis.

11. Identify each statement as true or false, and give a reason for your choice.

a. The decision to reject or not reject the null hypothesis is based on the size of the *p*-value.

**Sample Answer:** True. Small *p*-values indicate a low probability that an observed sample mean would occur by chance from the hypothesized population.

b. If the null hypothesis is true, a *p*-value of 0.05 would mean that on average one out of 20 samples would result in a mean at least as big as that observed in our random sample just by chance.

**Sample Answer:** True, provided that $H_a$ is a “greater than” hypothesis. This is the meaning of a *p*-value.

c. If a *p*-value is 0.04 and $H_a$ is a "greater than" hypothesis, there is a 96% chance that the sample mean you observed is from a sample from a population with a mean equal to the null hypothesis.

**Sample Answer:** False. Since the sample has already been selected, it is nonsense to speak of the probability that it ***will*** have any particular property. It is what it is. What you can say is that future random sampling from the population would lead to sample means smaller than you observed in 96% of those future samples.

d. Small *p*-values suggest that the null hypothesis is unlikely to be true.

**Sample Answer:** True. The smaller the *p*-value is, the more convincing is the rejection of the null hypothesis. It indicates the strength of evidence for rejecting the null hypothesis.

e. In working with *p*-values, you always begin with a set significance level.

**Sample Answer:** False. Significance is associated with alpha levels set by the researcher before sampling occurs; *p*-values always begin from calculating an observed sample statistic.

f. An observed sample mean can be significant at one alpha level but not significant at another.

**Sample Answer:** True. An observed sample mean might generate a critical value that falls between $\alpha$ = 0.05 and $\alpha$ = 0.01, which would be significant at the 5% level but not at the 1% level.

**TI-Nspire Navigator Opportunity: *Quick Poll***
**See Note 5 at the end of the lesson**.

## Wrap Up

Students should:

- Interpret a *p*-value as the probability that the means of future samples from the hypothesized population will be at least as extreme as the mean of the observed sample, provided the null hypothesis is true.
- Understand the connection between a *p*-value and the null hypothesis.

- Recognize that the size of the sample drawn from a given population will affect the magnitude of the *p*-value. In general, for sample means giving *p*-values less than 0.5, increasing the sample size decreases the *p*-value.

## TI-Nspire Navigator

**Note 1**

**Question 2, Name of Feature: Screen Capture**

This is an opportunity to use Screen Capture to display the different samples students have generated and to discuss how their answers to Question 2 part a might be different because of the variability in their samples

**Note 2**

**Question 3c, Name of Feature: Screen Capture**

A Screen Capture could be used to discuss why those with small p-values might believe their observed sample mean was unlikely to have occurred by chance.

**Note 3**

**Question 5c, Name of Feature: Screen Capture**

This is a good place to use the Navigator *Screen Capture* so the class can see the different p-values and the different simulated sampling distributions of the sample means. Stress that p-values will vary from student to student since p-values come from the sample, not from outside considerations.

**Note 4**
**Question 7, Name of Feature: Screen Capture**

A *Screen Capture* will show the different results for 100 samples. Looking at how these results are actually similar across the class can be a good opportunity to make very clear what p-values measure and how to interpret them by looking across different sample means that occurred by chance. You might make a student with an interesting p-value a Live Presenter to show his or her screens for 2.1 and 2.2 and discuss how these pages relate to his or her answer to Question 7.

**Note 5**
**Question 11, Name of Feature: Quick Poll**

Use *Quick Poll* to check student responses to Question 11 and organize class discussion based on those; i.e., Why do you think some people said true for part e?