Cramer's Rule

ID: 8793

Time required
30-35 minutes

Activity Overview

In this activity, students are introduced to determinants of two-by-two matrices by discovering for themselves that the determinant can be found by subtracting the products of the diagonals. Students are then introduced to Cramer's Rule and use it to solve three systems that have one solution, no solutions, and an infinite number of solutions, respectively. They graph the equations to verify their answers and are finally asked how Cramer's Rule can be used to determine the number of solutions to a system of linear equations.

Topic: Linear Systems

- *Classify the solution set of a pair of linear equations (in two variables) as a point or a line.*
- *Solve a system of two linear equations (in two variables) graphically.*
- *Solve a system of two linear equations (in two variables) using matrices.*

Teacher Preparation and Notes

This activity is designed to be used in an Algebra 2 classroom. Students should be familiar with solving systems of linear equations from Algebra 1, although you may wish to review that the solution is the point(s) of intersection of the graphs of the equations.

- *Students should already be familiar with the basic concepts of matrices.*
- *Students should know that division by zero is undefined and zero divided by any nonzero number is zero.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8793" in the keyword search box.***

Associated Materials

- *CramersRule_Student.doc*
- *CramersRule.tns*
- *CramersRule_Soln.tns*

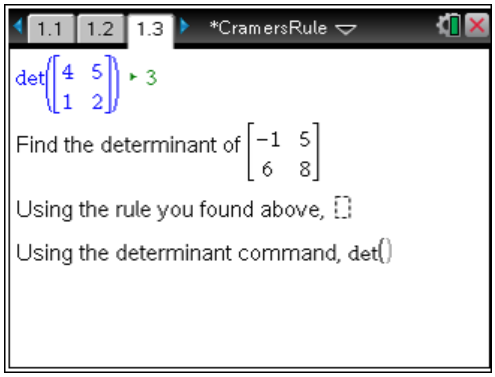
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Matrix Inverses (TI-Nspire technology) — 16031*
- *Matrix Multiplication (TI-84 Plus family) — 2128*
- *Solving Systems Using matrices – The Long way and the Short Way (TI-84 Plus family) — 8216*
- *Operating on Matrices (TI-Nspire technology) — 11357*

Problem 1 – Determinants

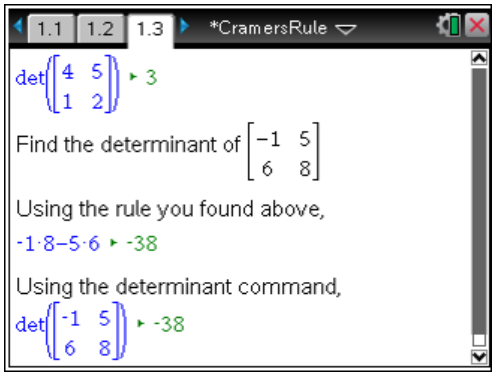
A blue, two-by-two matrix and its determinant are shown on page 1.3. Let students discover the rule to calculate a determinant by changing different entries in the matrix and observing the changing values of the determinant.



To change an entry, students should move the cursor over the entry and click on that value. Then press ↵ and type the new value. When all values have been changed, press enter for their calculated result. Remind them to use negative values and zero as well. Students should see that the determinant is the difference in the product of the diagonals:

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

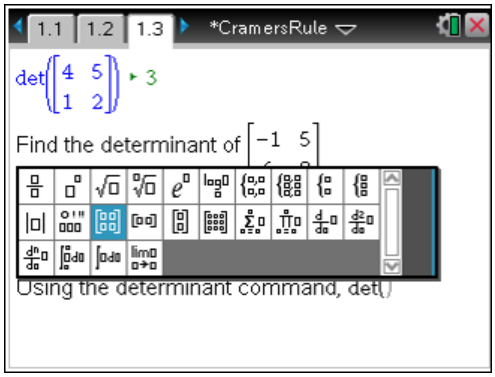
Once students know the rule, they should advance to the bottom of the page to calculate $\det\begin{bmatrix} -1 & 5 \\ 6 & 8 \end{bmatrix}$ by using their rule. The students need to click inside the first math box to enter their calculations and then press enter for their result. The students should then check their answer by using the **Det(** command in the second math box. To enter their matrix in this math box, click inside of it and then open the Math Templates (). A two-by-two matrix template is located in row two, column three. They should enter the value for each entry and press enter when complete. (Press tab to move through the entries.)



TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 1 at the end of this lesson.

Help students practice finding determinants on this page by challenging them to create matrices with determinants of zero, one, and one-half. See if any students created a matrix with a determinant of zero such that none of the entries are zero. Have these students share how they came about their answers



TI-Nspire Navigator Opportunity: *Screen Capture*

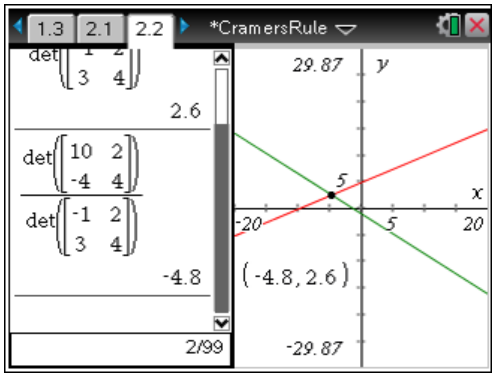
See Note 2 at the end of this lesson.

Problem 2 – Cramer's Rule

Introduce Cramer's Rule:

For a system of two equations written in standard form, $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, the coefficient matrix is the matrix of the coefficients: $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, and the determinant of the coefficient matrix is represented by *D*. Cramer's Rule states that the solution of the system is (*x*, *y*) where $x = \dfrac{\det\begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}}{D}$ and $y = \dfrac{\det\begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}}{D}$.

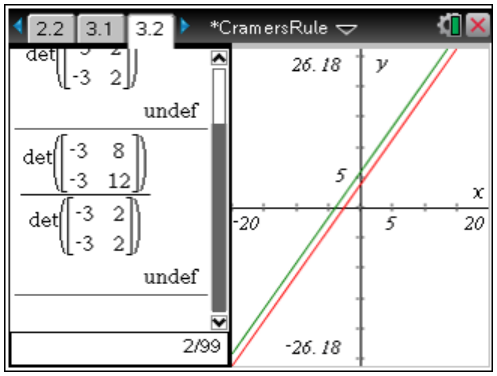
On page 2.2, students will use Cramer's Rule on the left side of the page to solve the system of equations found on page 2.1. To obtain decimal answers as those shown to the right, students must press ctrl enter.

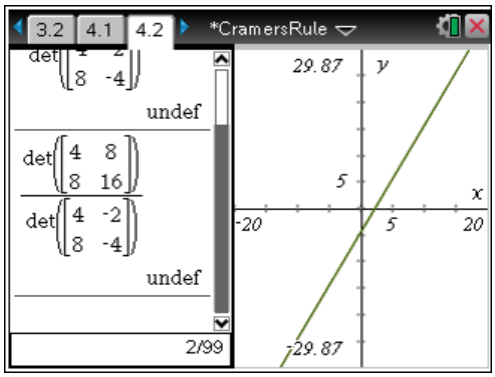


TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 3 at the end of this lesson.

Once students have the solutions, they should move to *Graphs* application and enter each equation into the Function Entry Line. (They will need to adjust the equations so that *y* is isolated.) Students can find the coordinates of the intersection by pressing **MENU > Points & Lines > Intersection Point(s)** and selecting each line. Have then repeat this process on pages 3.2 and 4.2 for the systems shown on pages 3.1and 4.2.





Finally, have a discussion on how Cramer's Rule can be used to determine if a system of linear equations has one, zero, or infinitely many solutions.

TI-Nspire Navigator Opportunities

Note 1

Problem 1, Quick Poll

Have students submit the rule they found. This should allow for class discussion and dispel any issues with subtracting the products found in the reverse order.

Note 2

Problem 1, Screen Capture

Use screen capture of the entire class and share results when prompted to find matrices whose determinants are zero. Discuss patterns that students might be able to conjecture.

Note 3

Problem 2, Screen Capture

Use screen capture to verify students are entering the correct values into the table and answering the questions correctly.