EXPLORATIONS

Activity 3

Repeating Elevens

Concepts/Skills:

Patterns, problem solving

Calculator:

TI-30Xa SE or TI-34

Objectives:

Students compute multiples of 11, 111, 1111, and so forth, search for patterns in the products, and write generalizations of those patterns.

Getting Students Involved

- Why is it important in doing mathematics to see patterns?

  Mathematics is the study of patterns. One goal in mathematics is to find generalizations.

Making Mathematical Connections

Ask the students the following questions:

- What can you tell me about the product of 11 times any single digit?

  The product is a two-digit number with the single digit repeated as the tens and ones digits.

- What do you think the product of 11 and a two-digit number would look like?

  Answers will vary.

You may want to review the use of the [K] key.

➠ Transparency Master K: Repeat an Operation

Carrying Out the Investigation

If students are having trouble finding patterns, let them work in pairs or small groups and encourage them to talk to their partners about the products.

If students are having trouble writing generalizations, let them talk to their partners about the patterns they see in the products.

- What is changing in that pattern? What is remaining the same?
- How could you express that pattern in general terms or in symbolic terms?

Encourage students to minimize the number of keys they press. For example, in problem 1, **x 11** can be stored in memory by using the [K] key (on the TI-30Xa SE).

Making Sense of What Happened

Have students share the patterns they found and the generalizations they wrote. Special attention should be given to the language students use in stating generalizations.

Continuing the Investigation

Investigate the patterns if you multiply a two-digit number by 101.

Investigate the patterns if you divide a two-digit number by 101.

Solutions

1. 11 x 1 = 11
11 x 2 = 22
11 x 3 = 33
11 x 4 = 44
11 x 5 = 55
11 x 6 = 66
11 x 7 = 77
11 x 8 = 88
11 x 9 = 99
11 x 10 = 110
11 x 11 = 121
11 x 12 = 132
11 x 13 = 143

2. Each product is 10 times the number plus the number itself.

3. Answers will vary.

4. 11 x 18 = 198
11 x 19 = 209

5. AB x 11 = AB0 + AB

6. 14 x 111 = 1554
18 x 111 = 1998
24 x 111 = 2664
26 x 111 = 2886
34 x 111 = 3774
36 x 111 = 3996
52 x 111 = 5772
54 x 111 = 5994
74 x 111 = 8214
78 x 111 = 8658

7. Each product is 100 times the number plus 10 times the number plus the number itself.

8. Answers will vary.

9. 32 x 111 = 3552
41 x 111 = 4551
53 x 111 = 5883
90 x 111 = 9990
98 x 111 = 10878

10. AB x 111 = AB00 + AB0 + AB =

$$\begin{array}{r} AB00 \\ AB0 \\ +\ \ AB \\ \hline \end{array}$$

11. When you multiply 111 by any two-digit number with a digit sum less than 10, the thousands digit of the product is the same as the tens digit of the given number, the hundreds and tens digits of the product are the sum of the digits of the given number, and the ones digit of the product is the same as the ones digit of the given number.

When you multiply 111 by any two-digit number with a digit sum greater than 9, the pattern changes because of the need to "carry" the tens digit of the digit-sum.

12. 24 x 1111 = 26664
26 x 1111 = 28886
34 x 1111 = 37774
36 x 1111 = 39996
54 x 1111 = 59994
58 x 1111 = 64438

13. Answers will vary.

14. 14 x 1111 = 15554
44 x 1111 = 48884
63 x 1111 = 69993
61 x 1111 = 67771
84 x 1111 = 93324

15. Each product is the sum of 1000 times the number plus 100 times the number plus 10 times the number plus the number itself.

AB x 1111 = AB000
AB00
AB0
+ AB

16. Answers will vary.

17. 63 x 111 = 6993
63 x 1111 = 69993
63 x 11111 = 699993
64 x 111 = 7104
64 x 1111 = 71104
64 x 11111 = 711104
65 x 111= 7215
65 x 1111= 72215
65 x 11111= 722215
66 x 111= 7326
66 x 1111= 73326
66 x 11111= 733326
67 x 111= 7437
67 x 1111= 74437
67 x 11111= 744437

18. There are many patterns students might describe:

- Ones digit of each product is the same as the ones digit of the given number.
- Tens digit of each product is the same as the ones digit of the sum of the digits of the given number.
- Hundreds of the first product in each set is repeated more often in successive products.