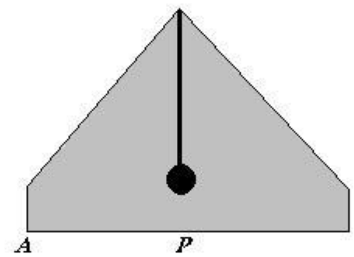
Simple Harmonic Motion

Student Activity

Name ___________________

Class ___________________

Problem 1 – Motion of a Swing



Imagine that you are on a swing at point *P*. Sit on the swing, push back to point *A* and then swing. Picture yourself moving back and forth.

Enter the equation $y = 2\pi \sin\left(\frac{x}{2}\right)$ in **y1** with $0 \leq x \leq 20$, $-10 \leq y \leq 10$ for the window settings. Select **Path** from the **F6:Style** menu. Now view the graph.

1. On the graph, when are you at point *A*? Point *P*?

2. Focus upon your speed, when are you moving fastest? When do you stop?

3. Now graph the derivative of **y1**. Enter ***d*(y1(x), x)** in **y2**. On this graph, when are you at point *A*? Point *P*?

4. Now focus upon the force acting on you. Once you start swinging, gravity pulls you downward. When is the acceleration the greatest? Least?

5. Graph the derivative of **y2**. Enter ***d*(y2(x), x)** in **y3**. On this graph, when are you at point *A*? Point *P*?

By Newton's Laws of Motion, force is directly proportional to acceleration. In fact, $F = m \cdot a$, where *m* is your mass, and *a* is your acceleration.

Since your mass is constant, the force you experience when on the swing is proportionally equivalent to your acceleration.

6. Motion is defined to be simple harmonic if acceleration is directly proportional to displacement from the origin. Clearly explain what this statement means.

The motion of the swing cyclic. Let *AP* = *A* (**amplitude**) and assume that $n = \frac{2\pi}{\text{seconds per cycle}}$. So the displacement equation for the swing can be written as $x = A \cdot \sin(nt)$.

7. Use the **displacement** equation to derive a formula for **velocity**.

8. From this equation, derive a formula for **acceleration** in terms of **time**.

9. Now, substitute the formula for **displacement** into this **acceleration** formula.

10. In your own words, explain how this formula relates to our motion on a swing.

11. Referring to the graphs of motion you have seen, carefully describe the critical points of this motion in terms of displacement, velocity, and acceleration.

Problem 2 – Extension

Use principles of projectile motion to derive the trajectory equation for the flow of water.

EX 1: Find other examples of simple harmonic motion and try to analyze these in the same way as we have for a child on a swing.

EX 2: Carefully study the following, then explain and justify each statement.

- $a = \frac{d^2}{dt^2}(x) = \frac{dv}{dt}$
- $a = v \cdot \frac{dv}{dx}$

EX 3: Show that $\int v \cdot \frac{dv}{dx} dx = \frac{1}{2}v^2$. Use these results to derive the simple harmonic motion (SHM) formulas.

EX 4: When might it be appropriate to express the time/displacement form using cosine instead of sine?