

# “Picking” Your Way Through Area Problems

By Judy Hicks

---

**ACTIVITIES  
eXCHANGE ID: 8270**


---

**Time required**  
45 minutes
 

---

## Activity Overview

In this activity, students will discover Pick’s Theorem by finding the relationship between area and the number of boundary points and interior points of a lattice polygon.

## Concepts

- Areas of polygons
- 

## Teacher Preparation

This investigation offers many possible extensions depending on the level of the student. The activity can be used in the middle grades or high school.

- Depending on the level and age of your students, you may first want to provide square dot grid paper or geoboards that contain many different polygons. Encourage students to find different ways of finding the areas of polygons or irregular shapes. After some exploration, challenge students to find a formula for area using boundary points and interior points by drawing polygons of the same area. With this part of the investigation, students should be able to skip to Problem 4. If some students are still struggling with the concept, invite them to work through the activity as is.
- Because the area formula is linear, the partial derivatives are constant. If the number of interior points is held constant, then the partial derivative of area with respect to boundary points is  $\frac{1}{2}$ . This can provide a hands-on example of derivatives.
- The screenshots on pages 22–24 demonstrate expected student results. Refer to the screenshots on page 25 for a preview of the student TI-Nspire document (.tns file).
- **To download the student .tns file and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “8270” in the quick search box.**

## Classroom Management

- This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds or computer software. Be sure to cover all the material necessary for students’ total comprehension.
- The student worksheet *GeoAct03\_PicksTheorem\_worksheet\_EN* is intended to guide students through the main ideas of the activity, while providing more detailed instruction on how they are to perform specific actions using the tools of the handhelds. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.
- Information for an optional extension is provided at the end of this activity. The extension provides students an opportunity to solve a system of equations using matrices.

## TI-Nspire™ Applications

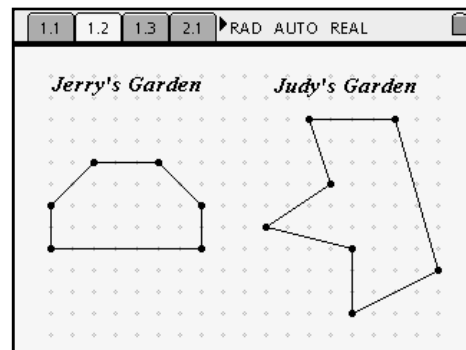
Graphs & Geometry, Lists & Spreadsheet, Notes

The focus question that defines this activity is this:

*What is the relationship between the area and the number of boundary and interior points of a lattice polygon?*

Some discussion should follow the posing of this question.

On page 1.2, students will see Jerry's garden and Judy's garden. Ask students to discuss different ways to determine the areas of the gardens (counting square units, dividing the garden into more manageable pieces, overlaying a rectangle around the entire garden, etc.).

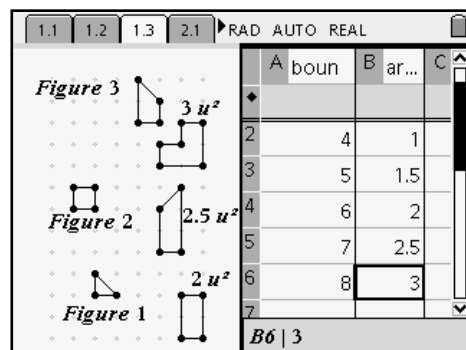


Page 1.2

**Problem 1 – Investigating polygons with zero interior points**

Have students look at page 1.3 and fill in the areas for the other polygons with different boundary points. (Students may need help with the vocabulary of *boundary* and *interior points*.) If students are using the handheld, remind them to use **ctrl** + **tab** to move between the applications in a split screen.

Encourage students to answer questions about polygons with no interior points. Students construct their own polygons by selecting the **Polygon** tool (**MENU > Shapes > Polygon**). Press **enter** on each grid point, and return back to the first vertex point to finish the polygon. For this investigation, only construct polygons with zero interior points. To measure the areas of the polygons, students can use the **Area** tool (**MENU > Measurement > Area**).



Page 1.3

**Student Worksheet Solutions**

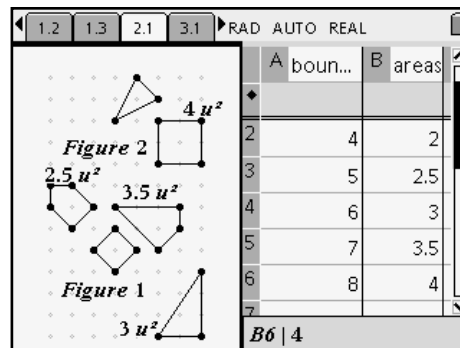
1. The area increases by 0.5 square unit for each boundary point added.
2. Answers will vary. Sample answer: The area is one less than half the number of boundary points.
3. Answers will vary but should reflect the answer for the previous question.

**Problem 2 – Polygons with one interior point**

Problem 2 shows polygons with one interior point. Again, have students investigate the polygons and fill in the spreadsheet with the appropriate areas. Students can also construct their own polygons if necessary, as instructed above. Solicit discussion about the questions that students should answer about area and what has changed with the adding of one interior point.

**Student Worksheet Solutions**

1. The area increases by 0.5 square unit for each boundary point added.
2. Answers will vary. Sample answer: The area is half the number of boundary points.
3. Answers will vary but should reflect the answer for the previous question.
4. Answers will vary. Sample answer: The formula no longer has a “– 1” in it.



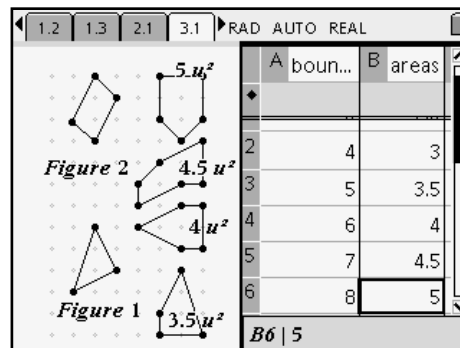
Page 2.1

**Problem 3 – Polygons with two interior points**

Lead students through Problem 3, which investigates polygons with two interior points. Students should be encouraged to cooperate with each other in answering the questions which follow Problem 3.

**Student Worksheet Solutions**

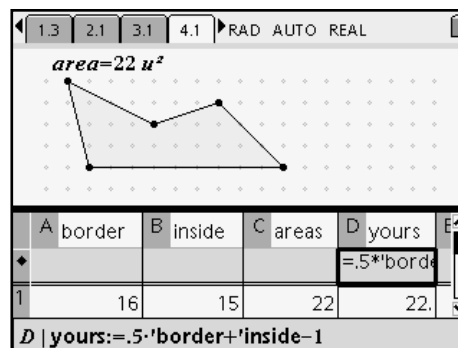
1. The area increases by 0.5 square unit for each boundary point added.
2. Answers will vary. Sample answer: The area is one more than half the number of boundary points.
3. Answers will vary but should reflect the answer for the previous question.
4. Answers will vary. Sample answer: The formula now has a “+ 1” in it.



Page 3.1

**Problem 4 – A polygon constructed to manipulate**

Go over Problem 4 with students. On page 4.1, students can manipulate the polygon by hovering over a vertex and pressing  $\text{ctrl} + \text{Ⓜ}$  to grab the polygon. Then use the NavPad to move the vertex to a different grid point, changing the number of boundary points, interior points, and area. Count the number of boundary and interior points, and record each value in the accompanying spreadsheet (Columns A and B). Have students enter the area of the figure in Column C. Have students analyze the formulas they created in Problems 1, 2, and 3. Help students formulate an area formula in Column D using the border and inside lists. When the students get the formula correct, the value in Column D should match the value in Column C. Students should come up with Pick's Theorem here. With students, go over the area of Jerry's and Judy's gardens using the new formula.



Page 4.1

**Student Worksheet Solutions**

1.  $A = 0.5 \times B + I - 1$ , where  $A$  is area,  $B$  is the number of boundary points, and  $I$  is the number of interior points.
2. Yes, Jerry was correct.
3. The area of Judy's garden is 36.5 square units.

**Extension**

Have students find the areas of three polygons as well as the number of interior points and the number of boundary points of each. Create the equation  $\text{Area} = a \times I + b \times B + c$  ( $I$  = interior,  $B$  = boundary). Substitute the values into three equations to get a system of equations to solve for the three coefficients  $a$ ,  $b$ , and  $c$ . This is a great way to introduce systems of equations and using matrices to solve a system of equations.



Visit [education.ti.com/exchange](http://education.ti.com/exchange) to download activity files, including the student .tns file *GeoAct03\_PicksTheorem\_EN.tns*. Enter "8270" in the quick search box.

1.1 1.2 1.3 2.1 ▸RAD AUTO REAL

**"PICKING" YOUR WAY  
THROUGH AREA PROBLEMS**

**Geometry**  
Finding areas of lattice polygons

1.1 1.2 1.3 2.1 ▸RAD AUTO REAL

*Jerry's Garden*      *Judy's Garden*

1.1 1.2 1.3 2.1 ▸RAD AUTO REAL

*Figure 3*

*Figure 2*

*Figure 1*

	A boun...	B ar...	C
1	3	.5	
2	4	1	
3	5	1.5	
4	6		
5	7		
6			

AI | 3

1.1 1.2 1.3 2.1 ▸RAD AUTO REAL

	A boun...	B areas
1	3	1.5
2	4	2
3	5	
4	6	
5	7	

AI | 3

1.2 1.3 2.1 3.1 ▸RAD AUTO REAL

	A boun...	B areas
1	3	2.5
2	4	3
3	5	
4	6	
5	7	

AI | 3

1.3 2.1 3.1 4.1 ▸RAD AUTO REAL

*area=6 u<sup>2</sup>*

	A border	B inside	C areas	D yours
1				

AI |

2.1 3.1 4.1 4.2 ▸RAD AUTO REAL

*Jerry's Garden*      *Judy's Garden*

*Was Jerry correct?*