Volume by Cross Sections

ID: 12281

Time Required
15 minutes

Activity Overview

*In this activity, students will be introduced to the concept of finding the volume of a solid formed by cross sections of a function that form certain shapes. Since volume is the area of the base times the height and dV = Area·dx, student review areas of various shapes like squares, semicircles, and equilateral triangles using self-check questions. 3D Parametric and **Geometry Trace** are used to help students get a "3D" visual of the volume being considered. Students will practice what they learn with exam-like multiple-choice questions.*

Topic: Volume by Cross Sections

- *Applications of integration*
- *Volume by cross sections*

Teacher Preparation and Notes

- *Part 1 of this activity takes less than 15 minutes. Part 2 contains three multiple-choice exam-like questions that have accompanying visual animations that can be used as an extension or homework.*
- *Students will write their responses directly into the TI-Nspire handheld and/or on the accompanying handout. On self-check questions, after answering the question students can press* [menu] *and select **Check Answer** (or* [ctrl] ▲*). If desired, by using the TI-Nspire Teacher Edition software, teachers can change these self-check questions to exam mode so students cannot check their answers. On any question, click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12281" in the keyword search box.***

Associated Materials

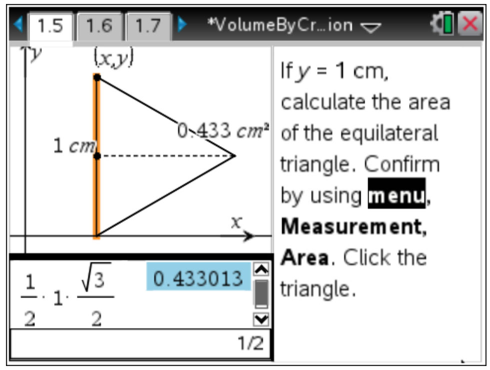
- *VolumeByCrossSections_Student.doc*
- *VolumeByCrossSections.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

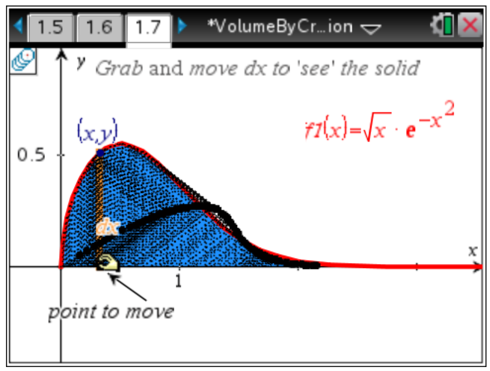
- *Solids of Revolution (TI-Nspire technology) — 17390*
- *Solids of Revolution Between Two Curves (TI-Nspire technology) — 17574*

Part 1 – Setting Up The Problem And Understanding The Concept



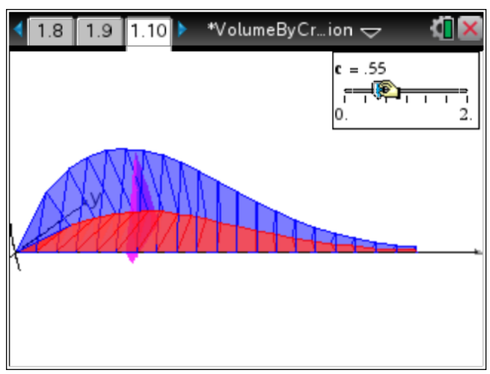
In this section students are introduced to the concept of finding the volume of a solid formed by cross sections of a function that form certain shapes. Since volume is the area of the base times the height and dV = Area dx, student review areas of various shapes like squares, semicircles and equilateral triangles.

Part 1 ends with students finding the volume of a solid with cross sections that are equilateral triangles.



Using **Geometry Trace** (**MENU > Trace > Geometry Trace**) on page 1.7 can give a visual similar to the one on the right. **Geometry Trace** requires that the students click (not grab) both the point and the triangle only once. They then can grab and move *dx*.

On page 1.10 there is a three dimensional model of the volume. Press **A** to auto rotate. Press **×** to zoom in. Press **÷** to zoom out. Other orientations can be quickly seen by pressing **X**, **Y**, **Z** or **O**.



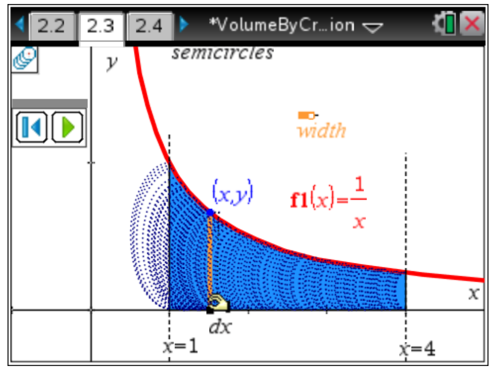
Student Solutions

1. *dx*
2. a. The area of a square with side x is x^2

   b. $\frac{1}{2}\pi r^2$
3. $\frac{1}{2}y\frac{\sqrt{3}}{2}y$
4. 0.433013 cm^2
5. Labeled (x, y) and the differential looks similar to the graph on page 1.7.
6. $$\int_0^2 \frac{1}{2}y\frac{\sqrt{3}}{2}y\,dx = \int_0^2 \frac{1}{2}\left(\sqrt{x}\cdot e^{-x^2}\right)\frac{\sqrt{3}}{2}\left(\sqrt{x}\cdot e^{-x^2}\right)dx$$
   $$= \int_0^2 \frac{\sqrt{3}}{4}x\cdot e^{-2x^2}dx$$
   If students use u-substitution, $u = -2x^2$, $du = -4x\,dx$ and the limits of integration are from 0 to –8.
   $$-\frac{\sqrt{3}}{16}\int_0^{-8} e^u du = -\frac{\sqrt{3}}{16}\left(e^{-8}-1\right) = \frac{\sqrt{3}}{16}\left(1-\frac{1}{e^8}\right)$$

Part 2 – Homework

This section enables students to get a visual of challenging exam-like multiple-choice questions. Question 1 and 2 are not calculator active; Question 3, with its decimal approximation answer, is a calculator-active question. Students should show their work on the first two questions and show their set up on the third question.



Student Solutions

1. (B) $\frac{3\pi}{32}$ units³
2. (B) 2 units³
3. (D) 1.57 units³