LESSON
4.3

PLANNING

LESSON OUTLINE

One day:

20 min	Investigation
5 min	Sharing
5 min	Example
15 min	Exercises

MATERIALS

- motion sensors
- graph paper
- Coordinate Axes (T), *optional*
- Sketchpad demonstration Lines, *optional*
- Calculator Note 4B

TEACHING

In this lesson students see how equations of lines change as the lines are translated.

One Step
Pose this problem: "What's an equation of the line that results from translating every point on the line $y = 2x$ right 3 units and up 5 units?" Encourage a variety of approaches.

During Sharing, introduce the term *translation* and encourage the class to look for patterns. Elicit the idea that all vertical translations of a line are horizontal translations, and vice versa; investigate together the question of how to determine what translation takes a line to itself.

INTRODUCTION

If necessary, remind students that a is the y-intercept, b is the slope, and (x_1, y_1) is a point on the line.

LESSON 4.3

Lines in Motion

In Chapter 3, you worked with two forms of linear equations:

Intercept form	$y = a + bx$
Point-slope form	$y = y_1 + b(x - x_1)$

In this lesson you will see how these forms are related to each other graphically.

With the exception of vertical lines, lines are functions. That means you could write the forms above as $f(x) = a + bx$ and $f(x) = f(x_1) + b(x - x_1)$. Linear functions are some of the simplest functions.

The investigation will help you see the effect that moving the graph of a line has on its equation. Moving a graph horizontally or vertically is called a **translation.** The discoveries you make about translations of lines will also apply to the graphs of other functions.

Free Basin (2002), shown here at the Wexner Center for the Arts in Columbus, Ohio, is a functional sculpture designed by Simparch, an artists' collaborative in Chicago, Illinois. As former skateboarders, the makers of *Free Basin* wanted to create a piece formed like a kidney-shaped swimming pool, to pay tribute to the empty swimming pools that first inspired skateboarding on curved surfaces. The underside of the basin shows beams that lie on lines that are translations of each other.

Investigation
Movin' Around

You will need

- two motion sensors

In this investigation you will explore what happens to the equation of a linear function when you translate the graph of the line. You'll then use your discoveries to interpret data.

Graph the lines in each step and look for patterns.

Step 1 On graph paper, graph the line $y = 2x$ and then draw a line parallel to it, but 3 units higher. What is the equation of this new line?

Step 2 On the same set of axes, draw a line parallel to the line $y = 2x$, but shifted down 4 units. What is the equation of this line?

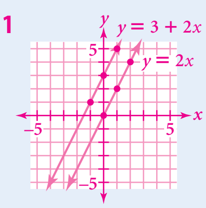
Step 3 On a new set of axes, graph the line $y = \frac{1}{2}x$. Mark the point where the line passes through the origin. Plot another point right 3 units and up 4 units from the origin, and draw a line through this point parallel to the original line. Write at least two equations of the new line.

Guiding the Investigation

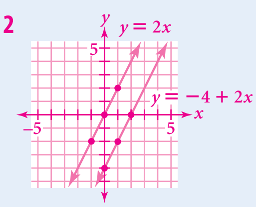
If you do not have motion sensors, completing Steps 1–5 is sufficient. If you have a few motion sensors, half the class can collect data for Steps 6–9 while the other half works on Steps 1–5. You might do Steps 6 and 7 as a demonstration. The data can be transferred to one calculator in each group.

Steps 1–3 As needed, remind students of how to find equations of lines given two points and how to find equations of lines parallel to another line.

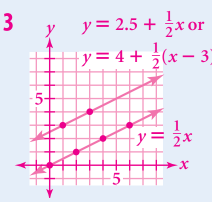
Step 1



Step 2



Step 3



Step 4 What happens if you move every point on $f(x) = \frac{1}{2}x$ to a new point up 1 unit and right 2 units? Write an equation in point-slope form for this new line. Then distribute and combine like terms to write the equation in intercept form. What do you notice?

Step 5 In general, what effect does translating a line have on its equation?

Your group will now use motion sensors to create a function and a translated copy of that function. [▶ See **Calculator Note 4B** for instructions on how to collect and retrieve data from two motion sensors. ◀]

Step 6 Arrange your group as in the photo to collect data.

Step 7 Person D coordinates the collection of data like this:

At 0 seconds:	C begins to walk slowly toward the motion sensors, and A begins to collect data.
About 2 seconds:	B begins to collect data.
About 5 seconds:	C begins to walk backward.
About 10 seconds:	A's sensor stops.
About 12 seconds:	B's sensor stops and C stops walking.

Step 8 After collecting the data, follow Calculator Note 4B to retrieve the data to two calculators and then transmit four lists of data to each group member's calculator. Be sure to keep track of which data each list contains.

Step 9 Graph both sets of data on the same screen. Record a sketch of what you see and answer these questions:

a. How are the two graphs related to each other?

b. If A's graph is $y = f(x)$, what equation describes B's graph? Describe how you determined this equation.

c. In general, if the graph of $y = f(x)$ is translated horizontally h units and vertically k units, what is the equation of this translated function?

Step 4 This movement creates the same line; $y = 1 + \frac{1}{2}(x - 2)$; $y = \frac{1}{2}x$; this equation is the same as your original line.

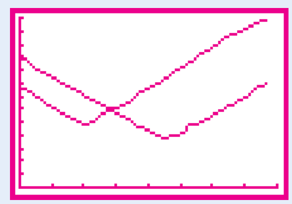
Step 5 The amount you translate to the right or left is subtracted from or added to x, and the amount you translate up or down is added to or subtracted from the function expression.

Steps 7, 8 You might want to demonstrate the setup in front of the class before students try this on their own.

While coordinating the collection of the data, person D may want to count off the seconds out loud so that all group members know when to start their assigned jobs.

Step 9 Encourage students to use a correct scale in their sketches. If a computer and printer are available with TI-Graph Link or TI-Connect, students can print a copy of their computer graph screen to attach to their investigation write-up.

Step 9



[0, 8, 1, 0, 13, 1]

Step 9a B's graph should be translated left about 2 units and up about 1 unit.

Step 9b $y = f(x + 2) + 1$, because B is delayed by 2 s and stands about 1 ft farther away from C.

Step 9c $y = f(x - h) + k$

Assessing Progress
In the investigation, you can observe students' facility with graphing parallel lines, finding equations of lines in point-slope form, and using motion sensors.

NCTM STANDARDS

CONTENT		PROCESS	
	Number		Problem Solving
✓	Algebra	✓	Reasoning
	Geometry	✓	Communication
✓	Measurement	✓	Connections
✓	Data/Probability	✓	Representation

LESSON OBJECTIVES

- Review linear equations
- Describe translations of a line in terms of horizontal and vertical shifts
- Write the equation of a translated line using h and k
- Understand point-slope form as a translation of the line with its equation written in intercept form
- Apply translations to functions