Parametric Equations Problems Sampler

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1. A dart is thrown upward with an initial velocity of 58 ft per second at an angle of elevation 41^{o}. Consider the position of the dart at any time t (t = 0 when the dart is thrown.) Neglect air resistance.
    a. Find parametric equations that model the problem situation.
    b. Draw a complete graph of the model.
    c. What portion of the graph represents the problem situation?
    d. When will the dart hit the ground?
    e. Find the maximum height of the dart. At what time will the dart reach maximum height?
    f. How far does the dart travel in the horizontal direction? Neglect air resistance.

2. An NFL punter at the 15-yard line kicks a football downfield with initial velocity 85 feet per second at an angle of elevation of 56^{o}.
    a. Draw a complete graph of the problem situation.
    b. How far downfield will the football first hit the field?
    c. Determine the maximum height of the ball above the field.
    d. What is the "hang time"? (That is, the total time the football is in the air.)

3. A batter hits a ball with an initial velocity of 92 feet per second and at an angle of 55^{o} from the horizontal. Find the maximum height attained and the total horizontal distance traveled by the ball.

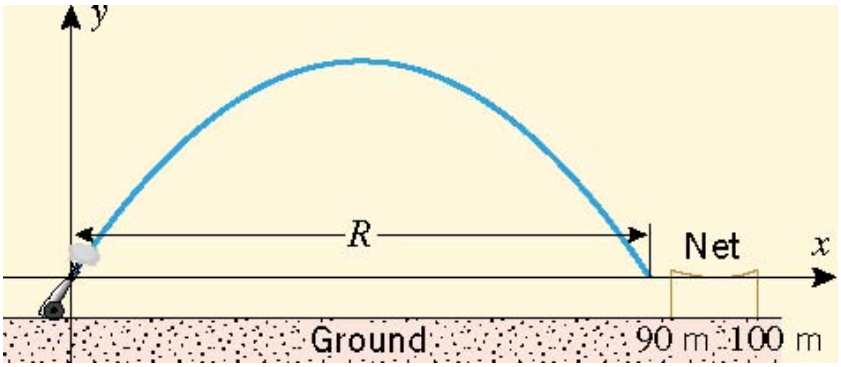
4. A golfer hits a ball with an initial velocity of 133 feet per second and at an angle of 36^{o} from the horizontal. Find when and where the ball will hit the ground. Will the ball clear a fence 9 feet high that is at a distance of 275 feet from the golfer?

5. It's the bottom of the ninth inning, the Cubs are behind 6-3 and the bases are loaded. Sammy Sosa is at bat. He swings and makes contact with the ball 3 feet above the plate at an angle of 20 degrees from the horizontal at a velocity of 150 feet per second. He hits straight toward center field where there is a fence 400 feet from home plate and 20 feet high. Neglect resistance due to wind. Does Sammy hit a grand slam and win the game?

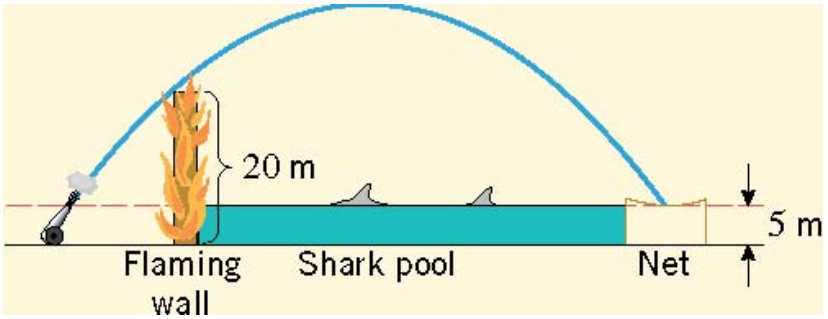
    a. Write equations and construct a complete graph to model the problem situation.
    b. Decide which is the more important factor: velocity or angle. Explain your choice with specific numerical examples using the above equations for a foundation. Use different values for angle and velocity in your determination. Take into account real-world issues of factors that affect velocity and angle.

6. Chris and Linda are standing 78 feet apart. At the same time, they each throw a softball toward each other. Linda throws her ball with an initial velocity of 45 ft per second with an angle of inclination of 44^{o}. Chris throws her ball with an initial velocity of 41 feet per second with an angle of inclination of 39^{o}.
    a. Find two sets of parametric equations that represent a model of the problem situation.
    b. Draw complete graphs of both sets of parametric equations in the same viewing rectangle.
    c. What values of t make sense in this problem situation?
    d. Find the maximum height of each ball. How far does each ball travel in the horizontal direction? When does each ball hit the ground? Whose ball hits first? Will the balls meet?

7. Blammo the Human Cannonball will be fired from a cannon and hopes to land in a small net at the opposite end of the circus arena. Your job as Blammo's manager is to do the mathematical calculations that will allow Blammo to perform his death-defying act safely. The methods that you will use are from the field of ballistics (the study of projectile motion). Blammo's cannon has a muzzle velocity of 35 m/s, which means that Blammo will leave the muzzle with that velocity. The muzzle opening will be 5 m above the ground, and Blammo's objective is to land in a net that is also 5 m above the ground and that extends a distance of 10 m between 90 m and 100 m from the cannon opening. Your mathematical problem is to determine the elevation angle of the cannon (the angle from the horizontal to the cannon barrel) that will make Blammo land in the net. Generate Blammo's trajectories, taking elevation angles at increments of 10° from 15° to 85°.



8. Blammo is to be fired from 5 m aboveground level with a muzzle velocity of 35 m/s over a flaming wall that is 20 m high and past a 5-m-high shark pool. To make the feat impressive, the pool will be made as long as possible. Your job as Blammo's manager is to determine the length of the pool, how far to place the cannon from the wall, and what elevation angle to use to ensure that Blammo clears the pool.



References

Memphis Educational Computer Connectivity Alliance
http://www.mecca.org/~halfacre/MATH/Parametric%20Equations%20Worksheet.pdf

Sosa problem adapted from Demana, Waits, Precalculus, A Graphing Approach, Menlo Park, Addison-Wesley, 1997, pp. 575-576.

Blammo problems adapted from a module by: John Rickert, Rose-Hulman Institute of Technology Howard Anton, Drexel University