NUMB3RS Activity: Close Encounters
Episode: "Under Pressure"

Topic: Social Networks, Means **Grade Level:** 9 - 12
Objective: Students will calculate a measure of "connectivity" to make decisions about the status of a network.
Time: 20 - 25 minutes
Materials: Paper and pencil

Introduction

In "Under Pressure," Don and his team are looking for a terrorist cell that is planning to poison the water supply in Los Angeles. Different groups within the cell are connected with other groups through lines of communication. These lines of communication are kept to a minimum so that the entire cell is not exposed if one group is discovered. Furthermore, to ensure additional secrecy, each group within a cell has knowledge of only their own role in the plot and is kept from knowing anything about the roles of the other groups.

Charlie explains that the structure of a terrorist cell is like a complex network. The network is made up of *nodes* (independent groups within a terrorist cell) and *edges* (connections between groups). He further states that the cell structure is dynamic and can shift and change over time. In this activity, students will study how to measure certain characteristics of a network and how changes in the network influence these measurements.

Discuss with Students

One goal of this activity is to help students make sense of the numerical measures they use to analyze the network. Ask the students to try to articulate why a smaller closeness measurement translates into a closer network. In Question 1, make sure that students see that the mean path length for a given group is found by taking the minimum distances from that group to the other five groups and dividing by 5. Some students may want to divide by 6 since there are 6 groups, but there are only 5 pairings. Try to get students to articulate why they think their networks for Question 7 minimize the characteristic path length differences when different groups are removed.

Social network analysis was introduced in the *NUMB3RS* activity "It's All Connected." The activity was written to introduce students to two measures of centrality: *degree* and *closeness*. This activity also introduces students to the closeness centrality but in a slightly different way. The closeness measure in this activity is the reciprocal of the one presented previously. This activity goes further to study how changes in a network affect the closeness measurement. To download "It's All Connected," go to **http://education.ti.com/exchange** and search for "6447."

Terry Wyberg, University of Minnesota, Minneapolis, MN

Student Page Answers:

1.

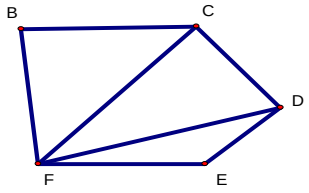
Shortest Distances between Groups

	A	B	C	D	E	F	Mean Shortest Distance for Each Group
A		1	2	2	2	1	1.6
B	1		1	2	2	1	1.4
C	2	1		1	2	1	1.4
D	2	2	1		1	1	1.4
E	2	2	2	1		1	1.6
F	1	1	1	1	1		1

2. *Group F has the lowest mean distance to each of the other groups. The lower the mean distance, the closer a group is to the other groups in the network.* **3.** *The characteristic path length for the group is 1.4.*

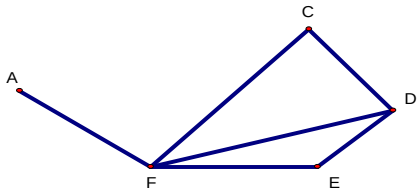
4.

a. Delete group A



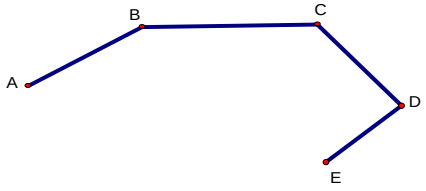
	B	C	D	E	F	Avg
B		1	2	2	1	1.5
C	1		1	2	1	1.25
D	2	1		1	1	1.25
E	2	2	1		1	1.5
F	1	1	1	1		1
characteristic path length =						**1.3**

b. Delete group B



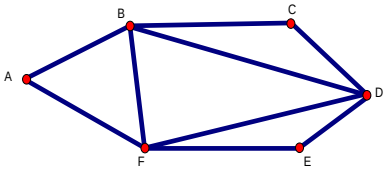
	A	C	D	E	F	Avg
A		2	2	2	1	1.75
C	2		1	2	1	1.5
D	2	1		1	1	1.25
E	2	2	1		1	1.5
F	1	1	1	1		1
characteristic path length =						**1.4**

c. Delete group F



	A	B	C	D	E	Avg
A		1	2	3	4	2.5
B	1		1	2	3	1.75
C	2	1		1	2	1.5
D	3	2	1		1	1.75
E	4	3	2	1		2.5
characteristic path length =						**2**

5. *The removal of group F causes the largest increase in the characteristic path length. Group F was connected to each of the other groups. Many of the shortest paths of the remaining groups ran through group F. The lengths of these paths will increase when F is removed.* **6.** *Yes, when group A was removed from the network the characteristic path length decreased. This happens because A was a remote group in the network.* **7.** *Answers will vary. The goal is to avoid hubs (group F) and to avoid isolated groups when a group is removed. Sample answer:*



Name: _____ Date: _____

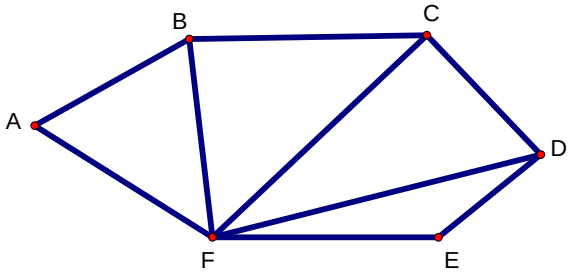
NUMB3RS Activity: Close Encounters

In "Under Pressure," Don and his team are looking for a terrorist cell that is planning to poison the water supply in Los Angeles. Different groups within the cell are connected with other groups through lines of communication. These lines of communication are kept to a minimum so that the entire cell is not exposed if one group is discovered. Furthermore, to ensure additional secrecy, each group within a cell has knowledge of only their own role in the plot and is kept from knowing anything about the roles of the other groups.

Charlie explains that the structure of a terrorist cell is like a complex network. The network is made up of *nodes* (independent groups within a terrorist cell) and *edges* (connections between groups). He further states that the cell structure is dynamic and can shift and change over time. In this activity, you will study how to measure certain characteristics of a network and how changes in the network influence these measurements.

Measuring Closeness

Suppose a cell is composed of six groups (A, B, C, D, E, F) connected as shown in the graph below. The edges represent direct lines of communication. The network needs to be connected so that each group can communicate with every other group.



Closeness measures how easily a group can communicate with each of the other groups in the network. One way to measure closeness for each group is to calculate the mean shortest distance to all the other groups in the cell. Distance in this example is the number of lines of communication needed to talk with another group. For instance, the shortest distance from A to B is 1 unit. The shortest distance from A to D is 2 units (A to F to D).

1. Determine the shortest distances between two groups, and calculate the mean shortest distance for each group.

Shortest Distances between Groups

	A	**B**	**C**	**D**	**E**	**F**	**Mean Shortest Distance for Each Group**
A		1	2	2	2	1	$\frac{1+2+2+2+1}{5} = 1.6$
B							
C							
D							
E							
F							

Terry Wyberg, University of Minnesota, Minneapolis, MN

2. Explain which group is the closest to all the others in the group. How does the mean shortest distance help determine this?

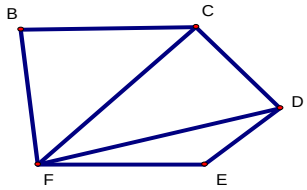
3. The *characteristic path length* for a network is the mean of all mean shortest distances for the groups. Determine the characteristic path length for the cell above.

Avoiding Hubs

A terrorist cell is designed to keep working smoothly even if one of the groups is removed from the network. If a dramatic increase in the characteristic path length occurs when a group is removed, then the cell is poorly designed, because the network communication is too dependent on one group.

4. Draw the network for each of the following situations starting with the network from Question 1 and determine the characteristic path length for each new cell.

a. Delete group A.



	B	C	D	E	F	Avg
B						
C						
D						
E						
F						
characteristic path length =						

b. Delete group B.

	A	C	D	E	F	Avg
A						
C						
D						
E						
F						
characteristic path length =						

Terry Wyberg, University of Minnesota, Minneapolis, MN

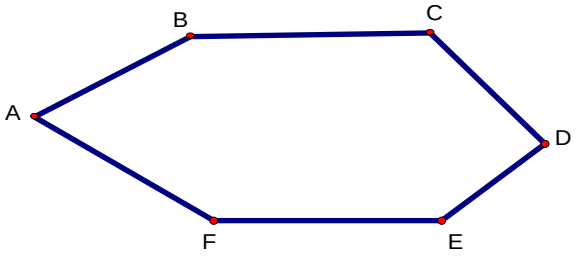
c. Delete group F.

	A	B	C	D	E	Avg
A						
B						
C						
D						
E						
characteristic path length =						

5. Which group causes the largest increase in the characteristic path length when removed? Explain why this happens.

6. Is it possible for the characteristic path length to decrease if a group is removed? Explain your answer.

7. Design your own network by adding 3 edges to the graph shown below. Add edges that would minimize increases in the characteristic path length if any one of the groups is removed. Explain why your cell design is less dependent on any one group than the network worked with in problems 1 to 5.



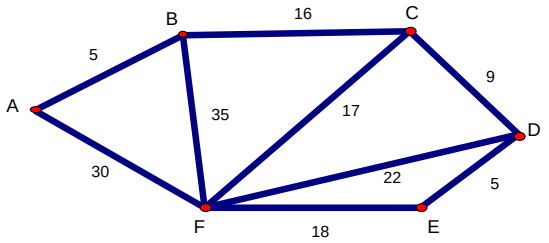
The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

- Create two networks with 4 groups (vertices) and 3 connections (edges). Create one that has the smallest characteristic path length and one that has the largest characteristic path length. The network needs to be totally connected (each group needs to be able to communicate with every other group).

- Create two connected networks with 5 groups (vertices) and 7 connections (edges). Create one that has the smallest characteristic path length and one that has the largest characteristic path length. The network needs to be connected.

- The lengths of the connections between any two groups in the networks described above were defined as a length of one unit. When Charlie further explains his network analysis, he mentions “weighted connections between vertices.” The connections between groups (vertices) could be defined as distances from one another or some other measure. The characteristic path length can also be used to study weighted networks. Repeat questions 1 to 5 with the network below. Assume that the lengths of the edges represent distances in miles that one group lives from the other (the figure is not drawn to scale).



Small World Networks

Another measure often used when studying small world networks is called a clustering coefficient. The clustering coefficient measures “cliquishness” of a network. This statistic is used along with the characteristic path length to study small world networks. To find out more information, go to: **http://people.ias.edu/~vazquez/publications/clustering.pre.1.pdf**

Related Activity

The *NUMB3RS* activity “It’s All Connected” is another activity on social networks. To download “It’s All Connected,” go to **http://education.ti.com/exchange** and search for “6447.”

Terry Wyberg, University of Minnesota, Minneapolis, MN