NUMB3RS Activity: Is This Seat Taken?
Episode: "Soft Target"

Topic: The Math of Arranging Objects **Grade Level**: 7 - 10
Objective: Introduce counting problems
Time: 20 - 30 minutes
Materials (optional): Different coins (pennies, nickels, dimes, and quarters)

Introduction

In "Soft Target," Alan is considering how to arrange people in seats at a wedding. Charlie points out that although there are many possibilities for seating arrangements, you reduce those possibilities if you decide who needs to sit next to whom.

Example If three people are to be seated on a bench, how many different ways could they be seated? The first person can sit anywhere. Once that person is seated, the next person has two choices of seats, and so on. Thus, there are 3•2•1 = 6 different ways the three people can be seated. The number of arrangements is written as 3! (3 factorial) and indicates a permutation of three people taken three at a time.

The Extensions explore seating arrangements leading to both Fibonacci and Fibonacci-like sequences. The Fibonacci sequence is 1, 1, 2, 3, 5, 8, … and can be represented recursively as: $a_1 = 1$

$$a_2 = 1$$

$$a_n = a_{n-2} + a_{n-1}.$$

A sequence similar to a Fibonacci sequence may have different initial or starting values as seen in the Extensions.

Discuss with Students

The simple seating problem above can be introduced and demonstrated in class. A tree diagram can show all 6 choices.

Encourage students to find a pattern by asking what if you had 5 people or 567 people to seat? Introduce the fundamental counting principle which states that if an event can occur in *p* ways and following this event, a second can occur in *q* ways, and following the second, a third can occur in *r* ways. The number of ways the events can occur in the order indicated is $p \bullet q \bullet r$ ways. The fundamental counting principle can be extended to any number of consecutive events

Explain to students that the number of seating arrangements possible and the rules for finding them change when you introduce other variables, such as restricting certain seats for certain people, or changing seating from a rectangular table to a circular one. Other seating arrangement possibilities will be explored in the student and extension pages.

Student page answers: 1. a. *24*; *check student's list.* **b.** *120* **c.** *n!* **2. a.** *2 and 3* **b.** *6* **3.** *120* **4.** *(n – 1)! or n!/n* **5. a.** *12* **b.** *48* **c.** *72* **Extension answers: First bullet**. 2, 3, 5, 8, … Each succeeding term is the sum of the two previous terms. **Second bullet**. 0, 2, 2, 4, 6, 8, … Each succeeding term is the sum of the two previous terms. **Third bullet**. 2, and 3.

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Name: ________________________________ Date: _______________

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Alan Eppes is helping to arrange the seating at a wedding and is overwhelmed by the number of ways to do it. Charlie helps him to see that there are a certain number of *unique* ways to seat people, and that if there are rules about who can sit next to whom, then there are even fewer ways to seat them.

1. a. Suppose there are four people who need to be seated in a row. You can use four different coins, a penny, nickel, quarter and dime, to represent the four different people. List all the ways you could seat them. Then, find how many ways they can be arranged in a row.

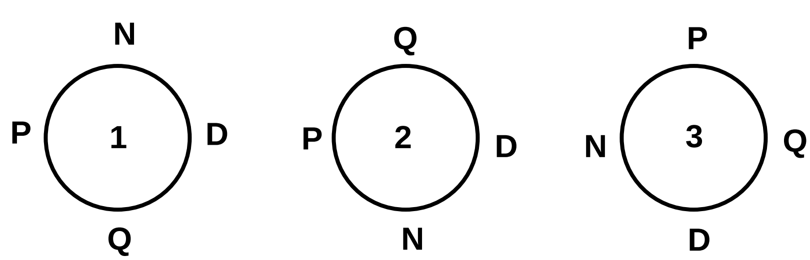
____
____
____
____

Look at your answers for **1a**. Rather than listing all of the ways to seat the people you can reach the same conclusions another way. For the first seat, any of the 4 people could sit there. For the second seat, you have 3 people left, so any of them could sit there. For the third seat, there are 2 people left, so either of them could sit there. For the fourth seat, there's only 1 person left, so that person has to sit there. 4•3•2•1 = 24 ways.

b. If there were 5 people to be seated in a row how many ways could they be seated? Explain how you know. ______________________________
____

c. If there were *n* people to be seated in a row how many ways could they be seated? Explain how you know. ______________________________
____

Suppose the four people from exercise **1** are seated at a round table. Below are three possible arrangements for them.



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2. **a.** Which two arrangements shown are considered the same arrangement? Explain your reasoning. ____
____
____

b. How many unique arrangements are there? ______________________

Compare your answers to exercises **1** and **2**. Clearly we cannot use the same logic to answer both exercises, but they are related. Seating four people in a row gives us 24 arrangements. When seating four people to a round table, the placement of the first person is required to determine the unique order of the rest of the table, so the number of arrangements possible is $3 \bullet 2 \bullet 1 = 6$. Another way to think about this is to realize that rotated arrangements are equivalent as long as people are sitting in the same relative location, so the number of arrangements possible is $\frac{24}{4} = 6$.

3. Six people are to be seated at a round table. How many different arrangements are possible? ____

4. Suppose there are *n* people (some unknown number) who need to be seated at a round table. How many different arrangements are possible? __________

Sometimes, special seating requirements can affect the ways that people are going to be seated. Married couples may need to sit together, for example, or people who do not get along should be separated.

5. Suppose that there are three bridesmaids and three groomsmen to be seated at a round table at the wedding.

a. If there must be a female between every two males, how many different ways can they be arranged? ______________________________________

b. Suppose that two of the bridesmaids are close friends who must sit together but no other special arrangements are made in the seating. In how many ways can the seats be chosen now? ______________________

c. Suppose that two of the bridesmaids once dated the same guy and really dislike each other. The two bridesmaids cannot sit together, but no other special arrangements are made in the seating. In how many ways can the seats be arranged now? ____

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

- Suppose at the wedding Alan arranges, both adults and children are attending. No one wants children to be disruptive at the wedding so no children can sit next to each other and no seat is left empty. If there are *n* seats in a row, in how many ways can you seat *n* adults and children? *[Hint: in a row with one seat, we could seat one adult or one child so there are 2 ways to fill the row. In a row with two seats, we could seat one adult and one child in either order for two ways, or two adults for a total of three different ways.]*
- If Alan's wedding arrangement is very informal, then one might insist that adults must not sit by themselves but next to another adult, and children must not sit by themselves but next to another child. If there are two or more seats, none are left empty. If there are *n* seats in a row, in how many ways can you seat *n* adults and children? *[Hint: if there is only 1 seat, it must be empty. If there are two seats, then there could be 2 adults, or 2 children for a total of 2 ways to fill the seats.]*
- The next version of the seating problem might be called family feud. No one wants to sit next to anyone. As a result, in a row of n seats, wherever there are two people in a row, there must be at least one seat empty between the two. How many ways could a row with one seat have (or not) have people seated? How many ways could a row with two seats have (or not) have people seated?

Additional Resources

Math Challenge #59:
http://www.figurethis.org/challenges/c59/challenge.htm
This challenge is designed for students who are in grades 5-10. The Fibonacci sequence shows up in unexpected places.

Fibonacci Series:
http://www.sciencenetlinks.com/Lessons.cfm?DocID=134
This site provides access to several different Fibonacci sequence activities.

Menage Problem:
http://math.dartmouth.edu/~doyle/docs/menage/menage/menage.html
For a more detailed explanation of some seating arrangements. This problem asks for the number of ways seating can occur at a round table with *n* couples with men and women alternating with no one sitting next to his or her partner.

Fibonacci Puzzles:
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibpuzzles.html#chairs
For more information on Fibonacci seating arrangements.

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