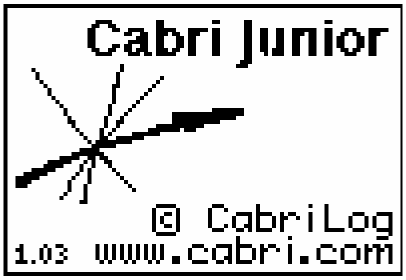
Activity 21 – The Pythagorean Theorem (Part 2)

First, turn on your TI-84 Plus and press the APPS key. Arrow down until you see Cabri Jr and press ENTER. You should now see this introduction screen.



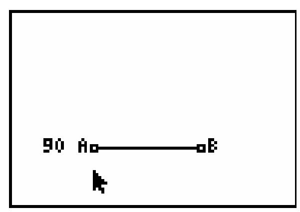
To begin the program, press any key. If a drawing comes up on the screen, press the Y= key (note the F1 above and to the right of the key – this program uses F1, F2, F3, F4, F5 names instead of the regular key names) and arrow down to NEW. It will ask you if you would like to save the changes. Press the 2nd key and then enter to not save the changes.

We are now ready to begin.

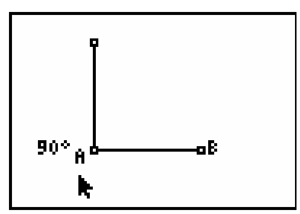
In this activity, we are going to look at a different proof of the Pythagorean Theorem. We hope to prove the statement that, if z is the hypotenuse of a right triangle and x and y are the legs of the right angle, then $z^2 = x^2 + y^2$.

This approach will require a bit of algebra work from Algebra 1, but don't worry, it won't be too painful!

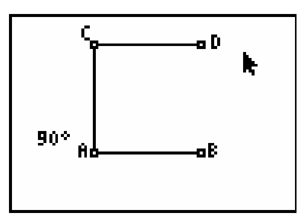
Let's start with a line segment AB. We need to rotate the line segment about point A, so add the label 90 to your sketch.



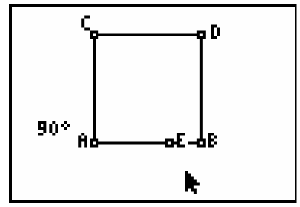
Access the rotation tool and click on point A as the center of the rotation, 90 as the angle of rotation and line segment AB.



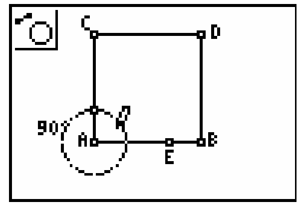
Label the new point as C. Continue by rotating line segment AC about point C through an angle of 90^{o}.



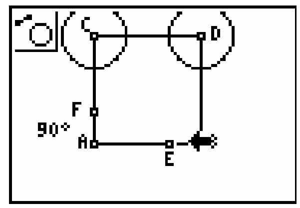
Complete the square by constructing line segment DB. Using the Point On tool, add point E on AB and construct line segment BE.



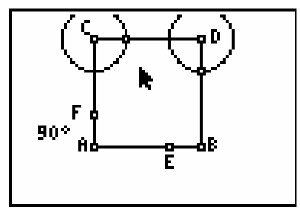
Using the Compass tool, construct a circle at point A with radius BE. Construct the point of intersection of this circle with line segment AC.



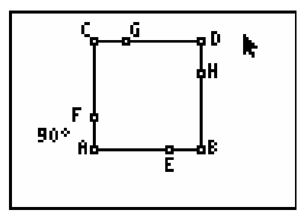
Label the point of intersection F. Using the Compass tool again to construct circles with centers at C and D and radius BE.



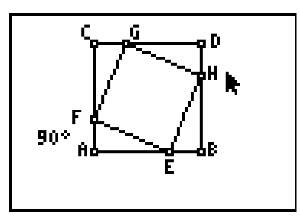
Construct the points of intersection of these circles with CD and DB.



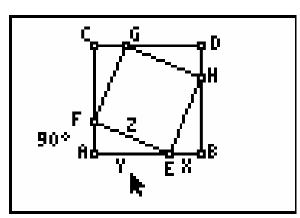
Label the new points of intersection as G and H.



Construct the quadrilateral EFGH. Can you prove that this quadrilateral is a square?



In this sketch, several segments have been labeled. BE has been labeled as x. In the same way, AF = CG = DH = x. Also, segment EA = y so FC = GD = HB = y. Since BACD is a square, each of the angles at A, B, C and D are 90^{o}, so we have four congruent triangles, namely EFA, FGC, GHD and HEB.



Now, let's look at the algebra in the situation. ABDC is a square with all sides of length $(x + y)$. The area of the square ABDC is:

$(x + y)^2$

$= x^2 + 2xy + y^2$

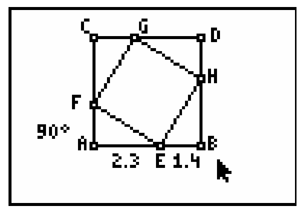
Each of the triangles EFA, FGC, GHD and HEB is a right angled triangle with height x and base y. So, the area of each triangle is ½ xy. The sum of the areas of the four triangles is 4* ½ xy = 2xy.

EFGH is a square with sides of length z. So the area of EFGH is z^2.

Looking at the areas in the diagram we can conclude that:

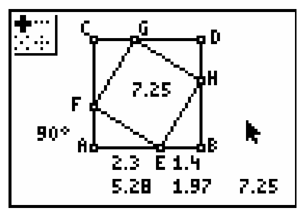
ABDC = AFE + FCG + GHD + HBE + EFGH	group the four triangles to get
ABDC = (AFE + FCG + GHD + HBE) + EFGH	substitute from above
$x^2 + 2xy + y^2 = 2xy + z^2$	subtract 2xy from each side
$x^2 + y^2 = z^2$	QED

Let's look at this numerically as well to confirm what we just proved algebraically. Measure EB and AB. Which other sides have the same measurements? Label each of them in your sketch.

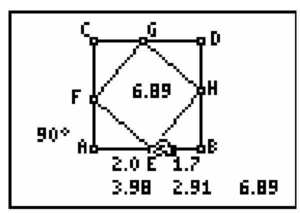


Find the squares of segments EB, AB and FE and use the Calculate tool to find the sum of the squares of the two segments AE and EB. In place of EB, consider the measurement as applied to segment AF. So, in the right triangle AFE, we have:

$EF^2 = FA^2 + AE^2$.



Drag point E to ensure that the relationship holds for other locations of the points E, F, G and H.



What would happen if you dragged one of points A or B? Would the relationship still hold?