EXPLORATIONS™

Activity 29

Equations of Circles

Objectives

- To investigate the relationship among the center, radius, and equation of a circle in the coordinate plane
- To investigate the relationship between the equation of a circle and the points inside, on, and outside a circle

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Introduction

Circles are commonly represented in the coordinate plane by an equation. In this activity, you will explore the relationships among the location of the center of the circle, the radius of a circle, and the equation of the circle in the plane. In addition, you will investigate the relationship between the points inside, on, and outside a circle and the equation of the circle.

Part I: Equation of a Circle

Construction

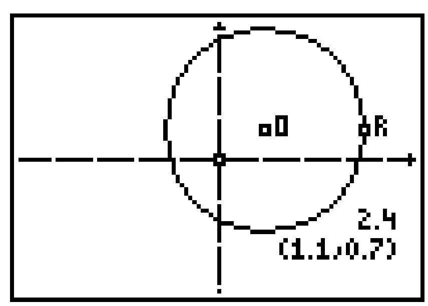
Construct a circle in the coordinate plane and identify its equation, center, and radius.

Show the axes on the screen. Move the origin to the center of the screen.

Draw a circle with a center *O* and radius point *R*, neither of which is attached to an axis.

Measure the radius of the circle by measuring the distance from point *O* to point *R*.

Display the coordinates of the center of the circle and the equation of the circle.



***Note:** Equation of the circle is not shown.*

Exploration

Investigate any relationships that exist between the radius, the coordinates of the center, and the equation of a circle. Be sure to observe these relationships as you move the entire circle as well as its defining points (*O* and *R*).

Questions and Conjectures

Make a conjecture about the relationship among the radius, the coordinates of the center, and the equation of the circle. Is this relationship true for all circles in the plane? Explain your reasoning.

Part II: Points Inside, On, and Outside a Circle

Construction

Adjust the construction to further investigate equations of circles.

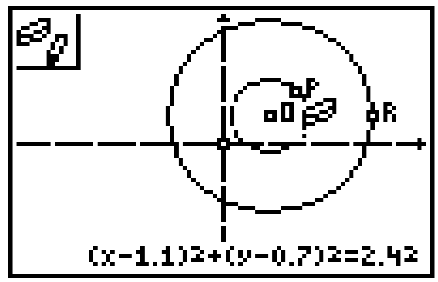
Continue using the previous construction.

Clear the coordinates of the center of the circle and the measure of the radius.

Construct a second circle with a center *O* and radius point *P* that is inside the first circle.

Show the equation of circle containing point *P*.

Hide the circle containing point *P*, but do not hide point *P*.



Note: *Equation of the new circle is not shown.*

Exploration

Observe the relationship between the equations of the two circles as you drag point *P* to different locations **inside** the original circle. Be sure to try this for different sizes and locations of your original circle.

Observe the relationship between the equations of the two circles as you drag point *P* to different locations **on** the original circle. Be sure to try this for different sizes and locations of your original circle.

Observe the relationship between the equations of the two circles as you drag point *P* to different locations **outside** the original circle. Be sure to try this for different sizes and locations of your original circle.

Questions and Conjectures

1. Make a conjecture about the relationship between the equation of a circle and the points inside, on, and outside the circle.
2. Do some research on the Distance Formula and explain the connections that this mathematical principle has with the circles and points investigated in this activity.

Teacher Notes

Activity 29

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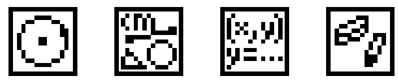
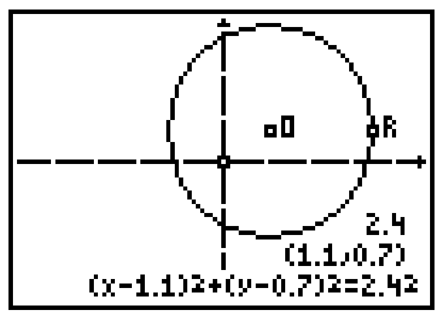


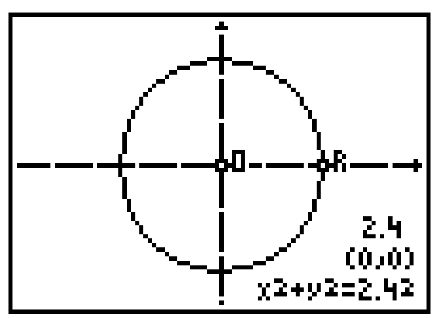
Part I: Equation of a Circle

Answers to Questions and Conjectures

Make a conjecture about the relationship among the radius, the coordinates of the center, and the equation of the circle. Is this relationship true for all circles in the plane? Explain your reasoning.



Students should discover that the equation of a circle is given by $(x - a)^2 + (y - b)^2 = r^2$ where (a, b) are the coordinates of the center of the circle and r is the radius of the circle. Dragging the circle around the plane helps students observe this relationship and solidify their own understanding of this standard form of the equation of a circle. As the circle is dragged around the screen, the coordinates of the center change while the radius of the circle remains constant. Changing the length of the radius of the circle will change the value of r^2 in the equation.



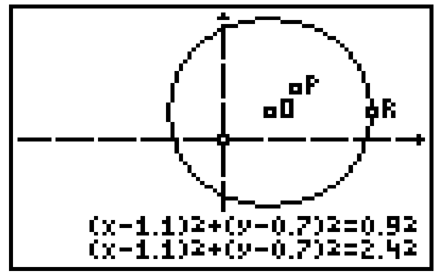
Placing the circle in the plane so that its center is at the origin or on the x- or y-axis will help students understand the special cases of this form of the equation of a circle.

Part II: Points Inside, On, and Outside a Circle

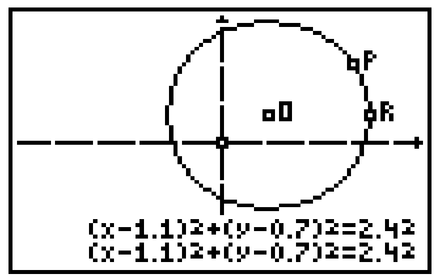
Answers to Questions and Conjectures

1. Make a conjecture about the relationship between the equation of a circle and the points inside, on, and outside the circle.

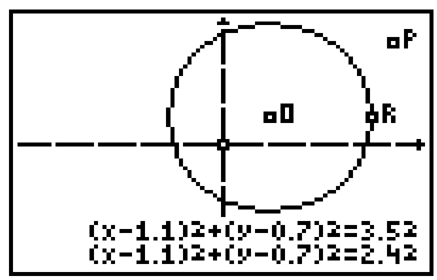
   When point P is inside the circle, the distance OP will be less than the radius of the original circle (distance OR). All of the points in the interior of the circle are represented by the inequality $(x - a)^2 + (y - b)^2 < r^2$ since the distance from O to P is less than the distance OR.

   When point P is on the circle, the distance OP will be equal to the radius of the original circle. All of the points on the circle are represented by the equation $(x - a)^2 + (y - b)^2 = r^2$ since the distance from O to P is equal to the distance OR.

   When point P is outside the circle, the distance OP will be greater than the radius of the original circle. All of the points in the exterior of the circle are represented by the inequality $(x - a)^2 + (y - b)^2 > r^2$ since the distance from O to P is greater than the distance OR.

2. Do some research on the Distance Formula and explain the connections that this mathematical principle has with the circles and points investigated in this activity.

   If students have studied the Distance Formula, they might see the connection between the equation of the circle and this formula. If they have not studied the Distance Formula, then draw their attention to the similarity between the equation of the circle and the computed relationship $(x_1 - x_2)^2 + (y_1 - y_2)^2$. When point P is on the circle, its coordinates, (x_1, y_1), dynamically represent all of the points on the circle. The coordinates of point O, (x_2, y_2), represent the center of the circle as they do in the equation of the circle. Using the Pythagorean Theorem, the expression $(x_1 - x_2)^2 + (y_1 - y_2)^2$ represents the square of the distance OP, or the square of the radius of the circle.

   This is a good time to repeat the locus definition of a circle: the set of points in a plane equidistant from a single point. This is the heart of the Distance Formula and the derivation of the general equation of a circle.