Getting to the Roots of Quadratics **STUDENT HANDOUT**

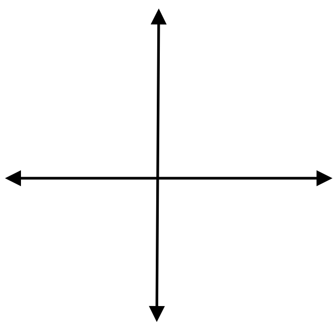
NAME ____

BACKGROUND

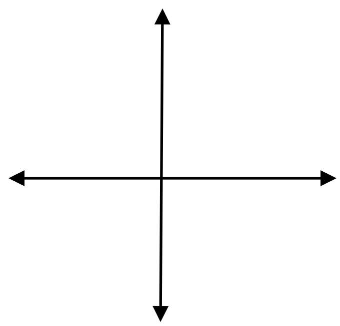
- **Graphically:** The real ***roots*** of a function $f(x)$ are the *x*-coordinates of the points at which the graph of the function intercepts/crosses the *x*-axis.

  For a quadratic function, whose graph is a parabola, what are the possibilities for the number of real roots? Sketch an example of each.

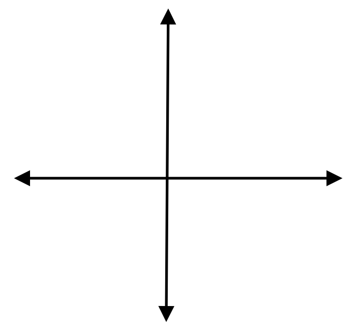
____ real root(s)



____ real root(s)



____ real root(s)



- **Algebraically:** The real roots of a function $f(x)$ are the solutions of the equation $f(x) = 0$, if there are any.

  For a quadratic function, $f(x) = ax^2 + bx + c$, we can find the solutions of $ax^2 + bx + c = 0$ using the ***quadratic formula*** to get:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

OVERVIEW

We will graph various quadratic functions $f(x) = ax^2 + bx + c$ where *a*, *b*, and *c* are rational numbers with $a \neq 0$ and investigate the nature of their roots. We will be asking the following types of questions:

- Does the quadratic have any real roots?
- If so, how many distinct real roots are there and are they rational or irrational? Can this be determined without finding the roots?
- How can we tell when there will be no real roots?
- The quantity, $b^2 - 4ac,$ is called the ***discriminant*** of the quadratic. Is there a connection between the discriminant and the nature of the roots of the quadratic?

Exploration

- **Open the TI-Nspire CAS document *RootsOfQuadratics*.**
- **Press ctrl ▶ to move to page 1.2.**
- **Read all the directions below before you begin working with page 1.2.**

On page 1.2, you will graph various quadratics, $f(x) = ax^2 + bx + c$.

- Change the values of *a*, *b*, and *c* by clicking twice on the text boxes to get a cursor inside the text box. Then use the clear key to delete the current value and type in the value you want. For now we will only use integer values for *a*, *b*, and *c*.

- Each time you change the values of *a*, *b*, and *c*, the corresponding quadratic $f(x) = ax^2 + bx + c$ will be graphed on the screen with its *x*-intercepts labeled (if there are any) and its corresponding value discriminant value, $d = b^2 - 4ac$.

- **GOAL:** Find graphs that have one *x*-intercept, two *x*-intercepts and no *x*-intercepts. That is, find graphs that have one (repeated/double) real root, two real roots, and no real roots.

  - ✓ Find **three** examples of each category: no real roots, one real root, two distinct real roots.

  - ✓ Record your examples in the Data Table below. For $x_1$and x_2, record the *x*-coordinates of any roots. If there is only one root, record that value for both x_1 and x_2. If there are no real roots, write “none” for x_1 and x_2.

  - ✓ The extra rows for Example 4 & Example 5 in the table for each category are to record any future data collection you might need to do later. Leave those blank for now.

  - ✓ Include examples with graphs that open up and well as down.

  - ✓ Try to include graphs that have integer *x*-intercepts as well as non-integer *x*-intercepts.

Data Table

	a	b	c	$d = b^2 - 4ac$	x_1	x_2
1 real root						
Example 1						
Example 2						
Example 3						
Example 4						
Example 5						
2 real roots						
Example 1						
Example 2						
Example 3						
Example 4						
Example 5						
No real roots						
Example 1						
Example 2						
Example 3						
Example 4						
Example 5						

Conjectures

Now look at the table of data you have collected above.

Fill in the blanks in the following conjectures:

- A quadratic has 1 real root when the discriminant $b^2 - 4ac$ is ________________.
- A quadratic has 2 real roots when the discriminant $b^2 - 4ac$ is ________________.
- A quadratic has 0 real roots when the discriminant $b^2 - 4ac$ is ________________.

Test Your Conjectures

On pages 1.3, 1.4, and 1.5 of the TI-Nspire file, check your conjectures by pressing tab until the icon beside the **Answer** portion of each screen is highlighted. Then press enter to reveal the answers.

In the table below,

- Find three new examples of values for a, b and c, one with a 0 discriminant, one with a positive discriminant and one with a negative discriminant.
- Be sure to calculate the discriminant to make sure it does what you want.
- Guess how many real roots $f(x) = ax^2 + bx + c$ has based on your above conjectures.

Discriminant	a	b	c	$b^2 - 4ac$	Guess how many real roots?
$b^2 - 4ac = 0$					
$b^2 - 4ac > 0$					
$b^2 - 4ac < 0$					

On page 1.2 in the TI-Nspire file,

- Change the values of the variables a, b and c to input your new three examples.
- Did your new quadratics give you the number of real roots you were expecting?
- Be sure to also record your new examples in the appropriate categories in the Data Table on page 3 of this worksheet.

Explanation

Let's now try to explain why the nature of the roots depends on the value of the discriminant.

Have you ever seen the expression for the discriminant, $b^2 - 4ac$, before? If so, where?

- On page 1.6 in the TI-Nspire file, check your answer.

- Page 2.1 in the TI-Nspire file is a page similar to that of page 1.2. Now however, the quadratic formula for the solutions of $ax^2 + bx + c = 0$ shows on the screen and changes as you change the values of a, b and c.

- Change the values of the variables a, b and c on page 2.1 to input the values in each of the 3 new examples you created on page 4 of this worksheet for the **Test Your Conjectures** section. Record the quadratic formula given for each of your 3 examples. In the last column, simplify the quadratic formula by hand to determine the *exact* values of the real roots if there are any. The screen on page 2.1 will give you decimal approximations of any roots in the coordinates of the *x*-intercepts.

Discriminant	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Exact Values of the Real Roots (if any)**
$b^2 - 4ac = 0$					
$b^2 - 4ac > 0$					
$b^2 - 4ac < 0$					

- Looking at the discriminants in the quadratic formulas in the table above, give explanations for the following:

Explain why the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ tells you that there is 1 real root for a quadratic $f(x) = ax^2 + bx + c$ when the discriminant $b^2 - 4ac = 0$. We say this is a ***double*** or ***repeated real root***. Write a formula for that one root in terms of a, b and c. Is this one root rational or irrational?

Explain why the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ tells you that there are no real roots for a quadratic $f(x) = ax^2 + bx + c$ when the discriminant $b^2 - 4ac < 0$.

Explain why the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ tells you that there are 2 (distinct) real roots for a quadratic $f(x) = ax^2 + bx + c$ when the discriminant $b^2 - 4ac > 0$.

Further Exploration: Positive Discriminants

Now we will focus on quadratics with positive discriminants. Look at your Data Table on page 3 of the worksheet for the values of a, b and c for which $b^2 - 4ac > 0$.

We already know that quadratics with positive discriminants will have 2 distinct roots that are real numbers. Real numbers include rational and irrational numbers.

Our goal is to make a conjecture about the types of positive values, $b^2 - 4ac$, which will produce real roots that are rational numbers. Remember that rational numbers are represented as terminating or repeating decimals. On our screen however, we may not be able to tell whether the roots are rational since we can only see 5 decimal places.

- Change the values of the variables a, b and c on page 2.1 to input the values of a, b and c from your Data Table on page 3 of the worksheet for which $b^2 - 4ac > 0$.
- Record the quadratic formula given on page 2.1 for each of your examples in the table below. Simplify the quadratic formula by hand to determine the *exact* values of the 2 real roots. The screen on page 2.1 will give you decimal approximations of any roots. Then classify the roots as rational or irrational. If all of your rational roots are actually integers, be sure to collect some other data that produces rational, non-integer roots as well.

Examples with Positive Discriminants	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	**Exact Values of the Real Roots**	**Rational or Irrational Roots?**
Example 1						
Example 2						
Example 3						

Examples with Positive Discriminants	a	b	c	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	**Exact Values of the Real Roots**	**Rational or Irrational Roots?**
Example 4						
Example 5						

Conjecture: Positive Discriminants
Complete the following conjecture:

The quadratic $f(x) = ax^2 + bx + c$ will have 2 distinct RATIONAL roots when the discriminant is

Test Your Positive Discriminant Conjecture

- Find two more examples of values for a, b and c with $b^2 - 4ac > 0$ which satisfy your above conjecture.

- Change the values for a, b and c on page 2.1 to input these two examples and record the quadratic formula for each example. Simplify it by hand to determine whether the two real roots whose decimal approximations are given on the screen are actually rational. Show your work below.

Getting to the Roots of Quadratics

Examples with Positive Discriminants	a	b	c	$b^2 - 4ac$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Exact Values of the Real Roots
New Example 1						
New Example 2						

Summary

Let's summarize our findings.

- The graph of a quadratic $f(x) = ax^2 + bx + c$ is called a ________________________.
- The roots of $f(x) = ax^2 + bx + c$ are the *x* values where the graph

  ____.
- The roots of $f(x) = ax^2 + bx + c$ are solutions of the equation

  ___________________________________.
- The nature of the roots depends on the discriminant, ____________________.
- If the discriminant is 0, $f(x) = ax^2 + bx + c$ has ______ real root(s). This root is given by

  x = ____________. This root is rational/irrational (circle the correct response).

  If the discriminant is positive, $f(x) = ax^2 + bx + c$ has ______ real root(s).

  In addition, if the discriminant is __________________, then the roots are rational. Otherwise they are irrational.

  If the discriminant is negative, $f(x) = ax^2 + bx + c$ has ______ real root(s).

Test Your Understanding

Answer the questions on pages 3.1-3.4. Check your answers.

Record your work for the questions on pages 3.5, 3.6 and 3.7 below. Check your answers on page 3.6.

1) $f(x) = 2x^2 + x - 3$

   $b^2 - 4ac =$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$

2) $f(x) = x^2 - x + 3$

   $b^2 - 4ac =$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$

3) $f(x) = x^2 - x - 3$

   $b^2 - 4ac =$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$

4) $f(x) = 4x^2 + 4x + 1$

   $b^2 - 4ac =$

   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$

Now check your answers to page 3.7.

Page 3.8 is a calculator page. Use the solve command to solve the equations $a \cdot x^2 + b \cdot x + c = 0$ for each of the four quadratics $f(x) = a \cdot x^2 + b \cdot x + c$.

The syntax is solve($a \cdot x^2 + b \cdot x + c = 0, x$).

This command will return "*false*" if there are no real roots.

Compare the answers you got with the ones given by the calculator. If there are any discrepancies, explain them or correct your work below.

Using the *solve* command	**Roots returned by handheld**	**Explanation of any discrepancies**
$solve(2x^2 + x - 3 = 0, x)$		
$solve(x^2 - x + 3 = 0, x)$		
$solve(x^2 - x - 3 = 0, x)$		
$solve(4x^2 + 4x + 1 = 0, x)$		

Great job getting to the roots of these quadratics!