

## Collecting Calvin's Trinkets

In portraying the richly imaginative world of Calvin & Hobbes, Bill Watterson has often described interesting mathematical situations. One particular strip introduced our heroes to the business world of marketing. Successful marketing campaigns are often built around collecting a complete set of *something*. For Calvin, it meant being "the first in his neighborhood to collect all ten colors" of plastic trinkets, each one found in a box of "chocolate-frosted sugar bombs." Calvin is eager to accept this challenge and he is also willing to accept the consequences of getting "pretty wired" after multiple bowls of cereal.

Let's consider the mathematics of Calvin's challenge. The fundamental question is "how many boxes of cereal does Calvin have to consume?" as he strives for his goal.

1. Perhaps he could accomplish this feat by buying ten boxes of sugar bombs.
  - a.) How likely is it that those first ten boxes would give him all ten colors?
  - b.) How many boxes do you think might be necessary to get all ten colors?
  - c.) Collect a guess from everyone in your class and determine a whole class average answer to the previous question.

Without buying lots of expensive cereal and getting "wired on sugar" the way Calvin does in his quest, we can try to determine mathematically how many boxes must be purchased in order for Calvin to collect all ten colors. A simulation is one way to figure this out. We will assume that the colors of the plastic trinkets have been distributed evenly in all of the cereal boxes, making each new box choice a random event.

2. We need to find a way to simulate the process of getting a randomly colored toy. Rolling a ten-sided die is one way to generate a random number, and each number could be assigned to a color. Each roll of the die would represent one cereal box purchased with its special plastic trinket inside. We should then record each roll of the die, taking particular notice of whether our "color number" is a *new* color or a repeat. We can stop rolling once we have collected a roll for each color. Counting the rolls would then tell us how many cereal boxes we had to purchase in order to get our first set of all ten colors. Would every child hoping to collect the complete set have to purchase that same number of boxes? Why or why not?

The TI-73 has a built-in "ten-sided die" that uses its ability to generate random numbers. Let's use this feature to perform the simulation of getting our first "set of all ten colors."

- Press **MATH** and arrow across to **PRB** and select **randInt(** which is the second option as shown. Press **ENTER** and then complete the command by typing the values shown at the right. You are asking the calculator to pick a random number between 1 and 10, like rolling a ten-sided die. The last value tells the calculator to do this one time when you press **ENTER**. Pressing **ENTER** again will repeat the process by “rolling” the die again.

```
MATH NUM PRB LOG
1:rand
2:randInt(
3:nPr
4:nCr
5:!
6:coin(
7:dice(
```

```
randInt(1,10,1)
```

- You can see that the first two rolls shown below have resulted in a gold toy and then a yellow one. Record your results on the Data Summary Sheet, entering a tally mark for each roll of the die. Continue recording until you have a least one tally mark in each space. Then count your rolls to determine how many “boxes” you had to purchase for your full set of all ten colors.

```
randInt(1,10,1)
(10)
randInt(1,10,1)
(3)
■
```

- Compare your results with your classmates. Determine who bought the most boxes and the fewest boxes. Find the class average number of boxes needed to get a full set of ten colors. We can think of that value as our experimental probability for the number of boxes needed.

Another classroom of students might arrive at a different average number. We can improve our estimate by repeating the simulation many times. The command **randInt(1,10,n)** can be altered to perform many rolls at once by changing the value of n. The tricky part is that we don't know initially how many rolls we might need to get a complete set of 10 colors. Using **randInt(1, 10, 50)** probably gives more rolls than we need, but we can ignore any extra rolls that occur after getting our last desired color.

- Using the list features of the TI-73, we can easily store our results and determine how many boxes are necessary for multiple sets. You can clear all lists by choosing **2nd**[STAT] and arrowing over to **OPS** and selecting the third option **ClrList**. Then press **LIST** and arrow up to the **L1** heading as shown at left. Press **MATH**, choose **PRB** and then **randInt(** as before and complete the statement as shown at the right, then **ENTER**. Your L1 list will now contain 50 rolls of the die for you to record. Scroll through the list, making tally marks below until you have gotten each “color” once. Be sure to

take note of **which entry** in the list gave you that tenth color. Remember that each classmate might have different stopping points. Why is that true?

L1	L2	L3	1
-----	-----	-----	
L1 =			

L1	L2	L3	1
-----	-----	-----	
L1 = ..nt(1,10,50)			

- Repeat this process, using L2 and L3 to store additional lists of 50 random numbers. Record your results in two of the charts on the Data Summary Sheet.

### Analyzing the Results

- Use L4 to gather classroom data, recording each N-value for members of your class. If 25 classmates each completed 3 different simulations of the event, there are 75 data points to enter. To do so, press 3 and move the cursor to L4(1) to enter the first value. The screen at the left shows 6 sample N-values, with the calculator is ready to enter the 7th data value. We can use the 7 points shown at the right to model the steps you can use to analyze your complete set of classroom data.

L2	L3	L4	4
2	1	25	
0	4	31	
0	1	34	
5	2	21	
1	2	26	
3	0	23	
L4(7) =			

L2	L3	L4	4
2	1	25	
0	4	31	
0	1	34	
5	2	21	
1	2	26	
3	0	23	
L4(1) = 25			

- Press [STAT] and arrow over to **MATH** to find some helpful statistics. Choose **min(** and then complete the command by pressing [2nd] [STAT] and choosing **L4** and [ENTER]. You can see that the minimum value in our sample set of seven values is 21 boxes purchased.

L3	OPS	MATH	CALC
1	min(		
2	max(		
3	mean(		
4	median(		
5	mode(		
6	stdDev(		
7	sum(		

min(L4)	21
---------	----

10. Press [STAT] and arrow over to **MATH** to find other functions. Choose **max()** and then complete the command by pressing [2nd][STAT] and choosing **L4** and [ENTER]. The maximum value in our sample set of seven values is 34 boxes purchased. The range is found by subtracting the max and min results.

```

Ls OPS MATH CALC
1:min(
2:max(
3:mean(
4:median(
5:mode(
6:stdDev(
7:sum(

```

```

min(L4) 21
max(L4) 34

```

11. Choosing **mean()** will allow us to calculate the mean for our list of data points in L4. The screen at the right shows that the mean for our seven sample boxes is about 28. How should we interpret this number for our simulation with Calvin's cereal boxes?

```

Ls OPS MATH CALC
1:min(
2:max(
3:mean(
4:median(
5:mode(
6:stdDev(
7:sum(

```

```

min(L4) 21
max(L4) 34
mean(L4) 28.42857143

```

L1	L2	L3	3
1	2	7	
2	2	4	
9	8	1	
10	4	7	
6	1	7	
4	3	7	
8	3	9	
L3() = 1			

### Digging Deeper into Theory

Our work has been experimental, determining a probability value by simulation. Now let's consider the theoretical value for the number of boxes we need to purchase to get all ten colors of toys. In general, we have a 1/10 chance of getting each color when choosing randomly. Another interpretation is that it might take us about 10 rolls to achieve a particular color number. So, if I want a red toy, I will sometimes get "lucky" and roll a 2 immediately, but it more likely that it will take longer. In fact, it actually takes on average 10 rolls, or the reciprocal of 1/10. As we think about the probability of getting a **new** color with each roll, it makes sense that it will get progressively more difficult when we get to the point of trying for our 8<sup>th</sup>, 9<sup>th</sup> and especially 10<sup>th</sup> new color.


12. The actual number of rolls needed is a sum of ten numbers, each representing the expected number of rolls needed to achieve a new color. On our first roll, we have 10/10 chance of getting a new color, so that number of rolls is exactly 1, a certainty. We can use  $P(1^{\text{st}} \text{ new color}) = 10/10$  to describe our probability, which then relates to "success" in exactly 1 roll. Then  $P(2^{\text{nd}} \text{ new color}) = 9/10$  and is also fairly easy, since we still have 9 colors to pick from. The number of rolls we would expect is the reciprocal of that or 10/9, also close to "success" in 1 roll.

Use the chart to summarize this process. Fill in the missing values to compute the theoretical value that answers the “how many rolls are needed” question.

New Color Number	Probability of Success	Reciprocal	Expected Number of Rolls for “Success”
1 <sup>st</sup>	10/10	1	1 roll
2 <sup>nd</sup>	9/10	10/9	1.111... ≈ 1 roll
3 <sup>rd</sup>	8/10	10/8	1.25 ≈ 1 roll
4 <sup>th</sup>			
5 <sup>th</sup>			
6 <sup>th</sup>			
7 <sup>th</sup>			
8 <sup>th</sup>			
9 <sup>th</sup>			
10 <sup>th</sup>			
	Total Rolls		

Explain why the “rolls for success” values get higher as we try for new colors in a shrinking list of possibilities.

13. The TI-73 can help us to add the fractions in the reciprocal column of our chart by using these keystrokes. To get the picture at the left, enter  $\boxed{1} \boxed{+} \boxed{1} \boxed{0} \boxed{\div} \boxed{9} \boxed{\triangleright}$  to make your screen match the left screen. Your cursor should be out of the denominator position and ready to insert the next addition sign to continue your sum. Record your sum of reciprocals in the last line of your chart.



The image shows a TI-73 calculator screen with the expression  $1 + \frac{10}{9}$  displayed. The '1' is on the left, followed by a plus sign, then '10' over '9' with a horizontal fraction bar. The cursor is positioned to the right of the fraction bar.

14. Compare the sum you found by adding the reciprocals to the experimental classroom mean that you arrived at in Step 11 for the number of boxes to be purchased. How would you explain this comparison to a friend in your class?

15. We generated "50 boxes" for each of our attempts at collecting a full set of colored trinkets. Is it possible that someone might have to purchase more than 50 boxes in order to collect a complete set?
  
16. Can you think of other "collecting situations" that can be solved by this same process? What aspects of the simulation process will remain the same and which might change with a new situation?

This strip was originally published in the newspapers on November 29, 1986.

Calvin shows Hobbes that you get a plastic trinket in the box of "Chocolate-Frosted Sugar Bombs" cereal. Hobbes notes the box says there are ten colors to collect. Calvin says Mom won't buy any more cereal until that box is gone. As they sit at the kitchen table with bowls in front of them, Hobbes figures it shouldn't take more than a couple hours to eat the box. Calvin isn't so sure, as he gets pretty wired after five bowls.

This strip was also published in the following books:

*Something Under the Bed is Drooling* on page 52

*The Essential Calvin and Hobbes* on page 178

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