Math Objectives

- Students will see how the solution to a system of linear inequalities is the intersection of each of the corresponding half planes.
- Students will see how the test point is used to verify the solution set.
- Students will understand that the solution regions can be one of four regions or no solution at all.
- Students will understand that the graph of a system of inequalities may or may not include parts of the boundaries as part of the solution.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- solution set
- boundary lines
- half plane
- linear inequality

About the Lesson

- Students begin by reviewing a graph of one linear inequality by testing a point and trying different shaded regions including solid or dotted boundary lines.
- Students then change the inequalities to get a particular region as the solution set to the system. This is followed up by testing a point algebraically.
- Students continue to look at other possible systems, including a system with a horizontal line and also a system with parallel lines.
- Finally, students find the inequalities to have a particular solution set. This is followed by testing a point algebraically.

TI-Nspire™ Navigator™

- Use Quick Poll questions to assess student understanding.
- Use Screen Capture to monitor student progress.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.

Lesson Files:

Student Activity
- Systems_of_Linear_Inequalities_1_Student.pdf
- Systems_of_Linear_Inequalities_1_Student.doc

TI-Nspire document
- Systems_of_Linear_Inequalities_1.tns

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Discussion Points and Possible Answers

**Tech Tip:** If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (ꀦ) getting ready to grab the point. Also, be sure that the word point appears. Then press `ctrl` + `x` to grab the point and close the hand ( Phill.

**Teacher Tip:** Because of the size of the TI-Nspire document, it may run slower on the handheld.

Move to page 1.2.

1. Page 1.2 is a review of finding a solution set to one linear inequality. Move the point around and notice when the word true or false appears. What do these words refer to in this context?

**Answer:** As the point is being moved around, the coordinates are substituted for $x$ and $y$ in the equation or inequality and the word true or false refers to the truth of the equation or inequality for those values of $x$ and $y$. Because this problem starts with a point at (6, 3) and an equation, the result is “false.” The moving of the point reinforces the idea of utilizing a test point to determine the solution.

**Teacher Tip:** Prior to this activity, students might use colored pencils to lightly shade the graphs of linear inequalities. Have students describe how this helps show the solution of the system.

2. Select another inequality symbol by clicking on the up or down arrows. Now move the point to a location where this inequality is true. Verify that the coordinates make the inequality a true statement. This point is said to satisfy the inequality.

**Answer:** Many answers are possible as long as the chosen point’s coordinates satisfy the inequality.

**Teacher Tip:** If students need more practice, it is possible to double-click on the inequality to change it.
3. Click the arrows to change the inequality to ≤ and then to ≥. What points satisfy these new inequalities that did not satisfy the inequalities in question 2?

**Answer:** The points on the line itself now satisfy these inequalities.

**Move to page 1.3.**

4. Two equations are currently graphed dividing the plane into four regions. Drag the point until “The Solution” shows and both equations display “True.” Because there is no shading yet, what does “The Solution” indicate?

**Answer:** This shows that when the system is equations only, two lines can intersect at one point, no points, or infinitely many points. This particular system is showing one point of intersection or “The Solution,” which is the point (-2, 1).

5. Click the top set of arrows until \( y \leq x + 3 \) is displayed. Click the bottom set of arrows until \( y > -x - 1 \) is displayed. Test the point by dragging it into all regions. Complete the table.

**Answer:** The completed table is below.

<table>
<thead>
<tr>
<th>Regions</th>
<th>Point</th>
<th>( y \leq x + 3 ) (True or False)</th>
<th>( y &gt; -x - 1 ) (True or False)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>(-7, -2)</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>Right</td>
<td>(4, 2)</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>Top</td>
<td>(-2, 5)</td>
<td>False</td>
<td>True</td>
</tr>
<tr>
<td>Bottom</td>
<td>(-2, -4)</td>
<td>True</td>
<td>False</td>
</tr>
</tbody>
</table>

6. Describe the solution set in which both inequalities are true.

**Answer:** the right region where \( y \leq x + 3 \) and \( y > -x - 1 \) are both shaded
Move to page 2.1.

7. a. Again, the equations have divided the plane into four regions. Click the arrows until the top region contains the solution set with two dotted boundary lines. What are the inequalities that you found? Algebraically verify your test point here.

**Answer:** $y > 2x - 5$ and $y > -4x + 1$

Answers will vary but might include:
- If using the point (2, 3):
  - $3 > 2(2) - 5$ true
  - $3 > -4(2) + 1$ true

b. What happens to the test point when it is moved to a boundary line?

**Answer:** The test point will no longer work because it has to be in the region, which does not include the boundary lines.

Move to page 3.1.

8. Click the arrows until the bottom region contains the solution set with two solid boundary lines. What are the inequalities that you found? Algebraically verify your test point here.

**Answer:** The two inequalities are $y \geq 4x + 6$ and $y \leq 0.5x - 1$.

9. What happens to the test point when it is moved to a boundary line?

**Answer:** Because the boundary lines are part of the solution set, the test point will still work as long as it is within the region.

Move to page 4.1.

10. Explain why there is a horizontal line.

**Answer:** It is a horizontal line because the equation is $y = -3$, which means that all points that belong to the line have a $y$-value of $-3$. 
11. How would you change the inequalities so that the bottom-right region is the solution set? What inequalities would have this solution set? Algebraically verify your test point here.

**Answer:** Answers may vary because the question does not specify if the boundary line should be dotted or solid.

- \( y \leq 0x - 3 \) or \( y < 0x - 3 \)
- \( y > -2x + 1 \) or \( y \geq -2x + 1 \)

Test point will vary.

**Move to page 5.1.**

12. Explain why the lines are parallel.

**Answer:** Parallel lines have equal slope.

**Teacher Tip:** This example can lead to a discussion of geometric applications with systems of linear inequalities. What other inequalities would be needed to create a parallelogram?

13. What inequalities would need to be used to have NO solution set?

**Answer:** There is no solution set when the area between the lines is not shaded. This occurs when one inequality is shaded up (\( y > x + 4 \) or \( y \geq x + 4 \)) and the other is shaded down (\( y < x - 1 \) or \( y \leq x - 1 \)).

**Move to page 6.1.**

14. The inequalities are not given on this page.

Determine the inequalities that determine the solution set. Algebraically verify your test point.

**Answer:** \( y \leq -x + 3 \) and \( y > 2x - 3 \)
Teacher Tip: Remind students of finding two points and finding slope and using the point-slope form of an equation.

15. Why must you use both inequalities to check a point in the solution region?

Answer: In order for a region to be the solution set, the test points must make both inequalities a true statement.

Teacher Tip: Teachers may want to continue this activity with inequalities like $x - 2y < 6$. Students often shade the wrong direction and without testing a point.

Wrap Up:
Upon completion of the discussion, the teacher should ensure that students understand:
- The solution set to a system of inequalities is where the shading of both inequalities overlap.
- All points within the solution set of a system of inequalities make both inequalities true.
- When boundary lines are not included as part of the solution set.

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Note 1
Question 2, Quick Poll or Screen Capture: You may want to send a Quick Poll to ensure students are able to find a true point. Ask students to identify the point they used, and use this to discuss the different points that are possible. Alternatively, you can scroll through screen captures and discuss the results.

Note 2
Question 11, Quick Poll: You may want to send a Quick Poll challenge to determine if students are able to distinguish between vertical and horizontal lines. Challenge students to write a system of inequalities such that the solution is bounded by a solid vertical line and a dashed horizontal line.